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SOME REMARKS ON RANDOM SERIES IN BANACH SPACES

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ABSTRACT. The aim of this note is to give: 1) a characterization of Banach spaces of cotype 2; 2) an example related to Rademacher series in the Schatten classes; 3) a characterization of finite dimensional Banach spaces.

INTRODUCTION

The study of convergence of random series in Banach spaces plays the key role in probability theory in Banach spaces and has a lot of important applications in functional analysis. Random series of a special form such as Gaussian, stable and Redemacher ones are the most interesting. Convergence of these series, in general, is related to some geometrical property of a Banach space. There are three mail results in this note. Theorem 1 of the note is a comparison theorem for convergence of Gaussian and stable series. It gives a characterization of Banach spaces of cotype 2. Theorem 2 concerns convergence of Rademacher series in well known Schatten classes S_p, $1 < p < 2$. Finally, we consider some properties of almost p-summing operators. Theorem 3 gives a characterization of finite dimensional Banach spaces.

1. DEFINITIONS AND NOTATION

Throughout this note we make use of the following notation:
$\mathbf{X}$ denotes a separable real Banach space, $\mathbf{X}^*$ denotes its dual space.
(ε_n) denotes a sequence of Bernoulli independent real variables, i.e., a sequence of i.i.d. real random variables with

$$P(\varepsilon_n = 1) = P(\varepsilon_n = -1) = \frac{1}{2}\,\cdot$$

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(γ_n) denotes a standard Gaussian sequence, i.e., a sequence of i.i.d. random real variables with characteristic function (c.f.) $e^{-t^2/2}$.
$(\gamma_n^{(p)})$ denotes a standard p-stable sequence, i.e., a sequence of i.i.d. random real variables with c.f. $e^{-|t|^p}$, $0 < p \leq 2$.

For our purposes we need the following geometric properties of Banach spaces.

Definition 1. A Banach space $\mathbf{X}$ is said to be of type p, $1 \leq p \leq 2$, if for any sequence (x_n) in $\mathbf{X}$ such that

$$\sum_{n=1}^{\infty} \|x_n\|^p < \infty,$$

the random series

$$\sum_{n=1}^{\infty} \varepsilon_n x_n \tag{1.1}$$

converges a.s. in $\mathbf{X}$.

A Banach space $\mathbf{X}$ is said to be of cotype q, $2 \leq q < \infty$, if for any sequence (x_n) in $\mathbf{X}$ such that the random series (1.1) converges a.s. in $\mathbf{X}$, we have

$$\sum_{n=1}^{\infty} \|x_n\|^q < \infty.$$

It is well-known that in this definition one can replace the Bernoulli sequence (ε_n) by the standard Gaussian sequence (γ_n). It is also known that the space L_r, $1 \leq r < \infty$, is of type $p = \min(2, r)$ and of cotype $q = \max(2, r)$. In particular, any Hilbert space is of type 2 and of cotype 2.

We refer the reader to [2, 3, 7, 16] for more information on probability in Banach spaces.

2. A characterization of Banach spaces of cotype 2

In [10] there is the following result:

Proposition 1. *Let (x_n) be a sequence of elements in $\mathbf{X}$ and r, p, q real numbers with $0 < r < p < 2$, $1/r = 1/p + 1/q$. If $(b_n) \in l_q$ and the random series*

$$\sum_n \gamma_n^{(p)} x_n$$

converges a.s. in $\mathbf{X}$*, then so does the random series*

$$\sum_n b_n \gamma_n^{(r)} x_n.$$

It is natural to raise the problem: *does hold true the preceding proposition for* $p = 2$*?* Let us begin with an example which says that, in general, the statement in the above proposition is not valid for $p = 2$. Indeed, consider, for instance, $\mathbf{X} = l_4$, $r = 1$, $p = 2$, $(a_n = 1/\sqrt{n})$, $(x_n = a_n e_n)$, where (e_n) is the natural basis in l_4. In this case $q = 2$ and the random series

$$\sum_n \gamma_n a_n e_n$$

converges a.s. in l_4 as $(a_n) \in l_4$ (according to Vakhania's Theorem, see [16, Theorem 5.6, p. 334]). On the other hand, if the random series

$$\sum_n \gamma_n^{(1)} a_n b_n e_n$$

converges a.s. in l_4 for any $(b_n) \in l_2$, we have:

$$\sum_n |a_n b_n| < \infty, \forall (b_n) \in l_2,$$

since any Banach space is of 1-stable cotype (see [8]). This implies that $(a_n) \in l_2$ which is impossible.

The full answer to the above problem is:

Theorem 1. *Let* $\mathbf{X}$ *be a Banach space and* r *a number with* $1 \leq r < 2$. *The following statements are equivalent:*
(a) $\mathbf{X}$ *is of cotype 2.*
(b) *The random series*

$$\sum_n b_n \gamma_n^{(r)} x_n$$

converges a.s. in $\mathbf{X}$ *for all* $(b_n) \in l_q$ *with* $1/r = 1/2 + 1/q$ *and any sequence* (x_n) *of* $\mathbf{X}$ *such that*

$$\sum_n \gamma_n x_n$$

converges a.s. in $\mathbf{X}$.

To prove this theorem we need the following result of Mushtari [9]. A probability measure μ in $\mathbf{X}$ is called Gaussian if every $x^* \in X^*$ (considered as a random variable from the probability space $(\mathbf{X}, \mu)$ into the real line) has Gaussian distribution. It is well-known that a symmetrical probability measure μ in $\mathbf{X}$ is Gaussian if and only if there is a sequence (y_n) in $\mathbf{X}$ such that μ is the distribution of the random series

$$\sum_n \gamma_n y_n$$

which is a.s. convergent in $\mathbf{X}$. In this case we have

$$\hat{\mu}(x^*) = \int_X \exp(i < x^*, x >) d\mu(x) = \exp\Big(-\frac{1}{2} \sum_n |\langle x^*, y_n \rangle|^2 \Big).$$

Let τ_μ denote the topology generated by the seminorm

$$\|x^*\|_\mu = \Big(\sum_n |\langle x^*, y_n \rangle|^2 \Big)^{1/2}.$$

By [9, Theorem 1] we have:

Lemma 1. *Let $\mathbf{X}$ be a Banach space of cotype 2 and μ a Gaussian measure in $\mathbf{X}$. If (x_n) is a sequence in $\mathbf{X}$ and r is a real number with $1 \leq r < 2$ such that*

$$\sum_n |\langle x^*, x_n \rangle|^r < \infty, \quad \forall x^* \in X^*,$$

and the function

$$\sum_n |\langle < x^*, x_n \rangle|^r$$

is continuous in the topology τ_μ, then there is a probability measure ν in $\mathbf{X}$ such that

$$\hat{\nu}(x^*) = \int_X \exp(i\langle x^*, x \rangle) d\nu(x) = \exp(-\sum_n |\langle x^*, x_n \rangle|^r).$$

Proof of Theorem 1.

(a) $\Longrightarrow$ (b). Let $\mathbf{X}$ be a Banach space of cotype 2. We must prove that the random series

$$\sum_n b_n \gamma_n^{(r)} x_n \tag{2.1}$$

converges a.s. in $\mathbf{X}$ for all $(b_n) \in l_q$ with $1/r = 1/2 + 1/q$ and $(x_n \in X)$ such that

$$\sum_n \gamma_n x_n \tag{2.2}$$

converges a.s. in $\mathbf{X}$. Indeed, the a.s. convergence of series (2.2) implies that $(\langle x^*, x_n \rangle) \in l_2$. On the other hand, by the Hölder inequality, we have:

$$\Big(\sum_n |\langle x^*, b_n x_n \rangle|^r\Big)^{1/r} \leq \Big(\sum_n |b_n|^q\Big)^{1/q} \Big(\sum_n |\langle x^*, x_n \rangle|^2\Big)^{1/2}.$$

This shows that the function

$$\sum_n |\langle x^*, b_n x_n \rangle|^r$$

is continuous in the topology τ_μ where μ is the distribution of the a.s. convergent series (2.2). Clearly, μ is Gaussian. By Lemma 1, there is a probability measure ν in $\mathbf{X}$ such that

$$\hat{\nu}(x^*) = \int_X \exp(i\langle x^*, x \rangle) d\nu(x) = \exp\Big(-\sum_n |\langle x^*, b_n x_n \rangle|^r\Big).$$

Consequently, series (2.1) converges a.s. in $\mathbf{X}$, by the Ito-Nisio theorem (see [4]).

(a) $\Longrightarrow$ (b). Let (x_n) be a sequence in $\mathbf{X}$ such that the series (2.2) converges a.s. in $\mathbf{X}$. To prove that $\mathbf{X}$ is of cotype 2 we must show that $(\|x_n\|) \in l_2$. As a matter of fact, by (b), we obtain that the random series

$$\sum_n b_n \gamma_n^{(r)} x_n$$

converges a.s. in $\mathbf{X}$ for all $(b_n) \in l_q$ with $1/r = 1/2 + 1/q$. It is well known that any Banach space is of r-stable cotype (see [8]), so that we have

$$\sum_n |b_n|^r \|x_n\|^r < \infty$$

for all $(b_n) \in l_q$, $1/r = 1/2 + 1/q$. Therefore, by Landau's theorem *, this implies that $(\|x_n\|) \in l_2$. The proof of Theorem 1 is complete.

* See G. H. Hardy, J. E. Littlewood, G. Polya, Inequalities, Cambridge Univ. Press, 1988, p. 120.

3. Rademacher series in the Schatten classes

Let (r_n) denote the Rademacher sequence (it is an example of Bernoulli sequences). It was proved in [11] that:

Proposition 2. *Let* $\mathbf{X}$ *be a Banach space,* H *a separable Hilbert space and* (e_n) *arbitrary fixed orthonormal basis of* H*. Consider the following statements:*
(a) $\mathbf{X}$ *is of a finite cotype (i.e.,* $\mathbf{X}$ *if of cotype* q *for some* q*,* $2 \leq q < \infty$*).*
(b) *For a continuous linear operator* $T : H \longrightarrow X$ *the random series*

$$\sum_n r_n T e_n$$

converges a.s. in $\mathbf{X}$ *if and only if* T^* *is absolutely 1-summing.*
Then (b) $\Longrightarrow$ *(a) is always true, and if in addition* $\mathbf{X}$ *is a GL-space,*
(a) $\Longrightarrow$ *(b) is also true.*

We now shall give an example which shows that, in general, *(a)* $\Longrightarrow$ *(b) is not true if* $\mathbf{X}$ *is not a GL-space* (see Theorem 2 below). To do this let us begin with well-known results on absolutely summing operators (due to Pietsch).

Definition 2. Let $\mathbf{X}$, $\mathbf{Y}$ be two Banach spaces and T a continuous linear operator from $\mathbf{X}$ into $\mathbf{Y}$. T is said to be an absolutely p-summing operator, $1 \leq p < \infty$, if $\sum\limits_n \|Tx_n\|^p < \infty$ for any sequence (x_n) in $\mathbf{X}$ such that $\sum\limits_n |\langle x_n, x^* \rangle|^p < \infty$, for all $x^* \in \mathbf{X}^*$.

The following is a corollary of Pietsch's domination theorem (see [12, 17.3.7, p. 234], and also [2, p. 44]).

Lemma 2. $T \in \Pi_2(l_2, \mathbf{Y})$ *if and only if* T *factors through* l_2 *in the form:* $T = BA$ *where* A *is a Hilbert-Schmidt operator from* l_2 *into* l_2 *and* B *is a linear continuous operator from* l_2 *into* $\mathbf{Y}$*.*

The following is (probably) known and we give the proof for the sake of completeness.

Lemma 3. *Let* $T : l_2 \longrightarrow l_2$*. Then* T *is a Hilbert-Schmidt operator if and only if* T *factors through* l_1*.*

Proof. It is known that T is a Hilbert-Schmidt operator if and only if for all $h \in l_2$ we have

$$Th = \sum_n \lambda_n \langle h, g_n \rangle f_n$$

where (g_n) and (f_n) are orthonormal bases in l_2, (λ_n) is a sequence of real numbers with $0 \leq \lambda_{n+1} \leq \lambda_n$ and

$$\sum_n |\lambda_n|^2 < \infty.$$

Put

$$A : l_2 \longrightarrow l_1$$
$$Ah = \Big(\lambda_n \langle h, g_n\rangle\Big),$$

$$B : l_1 \longrightarrow l_2$$
$$B(a_n) = \sum_n a_n f_n, \quad \forall (a_n) \in l_1.$$

Evidently, $T = BA$. The converse follows from the Grothendieck theorem which says that every continuous linear operator from l_1 into l_2 is absolutely 1-summing (see [2, p. 15]). $\square$

The following notion is taken from [13] (see also [2, p. 154 and 350]).

Definition 3. A Banach space $\mathbf{X}$ is said to be a GL-space if each absolutely 1-summing operator A from $\mathbf{X}$ into l_2 is absolutely 1-factorable, i.e. $A = CB$ where B is a continuous linear operator from $\mathbf{X}$ into some L_1 and C is a continuous linear operator form L_1 into l_2.

It is known that any Banach space with local unconditional structure is a GL-space (see [2]). In particular, Banach spaces with Schauder unconditional basis, Banach lattices are examples of GL-spaces. It is also shown that a Banach space $\mathbf{X}$ is a GL-space if and only if so is $\mathbf{X}^*$.

We now come to give examples of Banach spaces which are not GL-spaces. Let S_p, $1 \leq p < \infty$, denote the Schatten class in l_2. Recall that a continuous linear operator $A : l_2 \longrightarrow l_2$ belongs to S_p if and only if it can be represented in the form

$$Ah = \sum_n \lambda_n \langle h, g_n\rangle f_n,$$

where (g_n) and (f_n) are orthonormal bases in l_2, (λ_n) is a sequence of real numbers with $0 \leq \lambda_{n+1} \leq \lambda_n$ and

$$\sum_n |\lambda_n|^p < \infty$$

(see [2, p. 80]).

Put

$$\sigma_p(A) = \Big(\sum_n |\lambda_n|^p \Big)^{1/p}.$$

We need the following properties of S_p.

Lemma 4. 1) *Schatten's Theorem (see [14] or [2, p. 80-81]): (S_p, σ_p) is a Banach space for all p, $1 \leq p < \infty$ and the dual space of S_p is S_q with $1/p + 1/q = 1$.*
2) *Tomczak-Jaegermann's Theorem (see [15]): S_p is of type* $\min(2,p)$ *and of cotype* $\max(2,p)$.
3) *Pisier's Theorem (see [13] or [2, Theorem 17.24, p. 363]): If $p \neq 2$, S_p is not a GL-space.*

We now are going to prove the following:

Theorem 2. *For $1 < p < 2$ there exists a continuous linear operator $T : l_2 \longrightarrow S_p$ such that T^* is absolutely 1-summing, but the random series*

$$\sum_n r_n T e_n$$

does not converge a.s. in S_p, where (e_n) is the natural basis of l_2.

Proof. Let p, q real numbers with $1 < p < 2$, $2 < q < \infty$ and $1/p + 1/q = 1$. By the Lemma 4, S_p is of cotype 2 and S_q is not a GL-space for any $1 < p < 2$. Therefore, by the definition of GL-spaces, there is an operator $T : l_2 \longrightarrow S_p$ such that $T^* \in \Pi_1(S_q, l_2)$, but T^* can not factors through l_1. It is known that the random series

$$\sum_n r_n T e_n$$

converges a.s. in S_p if and only if $T \in \Pi_2(l_2, S_p)$, since S_p is of cotype 2 (see [11] or [2, Proposition 12.29, p. 251]). By Lemma 2, this implies that the operator T can be written in the form: $T = BA$, where $A : l_2 \longrightarrow l_2$ is a Hilbert-Schmidt operator and $B : l_2 \longrightarrow S_p$ is a linear continuous operator. Consequently, we have: $T^* = A^* B^*$, where $B^* : S_q \longrightarrow l_2$ and $A^* : l_2 \longrightarrow l_2$ is also a Hilbert-Schmidt operator. Therefore, A^* factors through l_1 (by Lemma 3), so that T^* factors through l_1 as well. Thus, we have got a contadiction. This ends the proof of Theorem 2. □

Remark. The idea of the above proof is borrowed from [6].

4. A Characterization of finite dimentional banach spaces

Let us begin with the following

Definition 4. Let $\mathbf{X}$, $\mathbf{Y}$ be Banach spaces and p a number with $1 \le p < \infty$. A continuous linear operator $T : \mathbf{X} \longrightarrow \mathbf{Y}$ is called almost p-summing if the random series

$$\sum_n r_n T x_n$$

converges a.s. in $\mathbf{Y}$ for all sequences (x_n) in $\mathbf{X}$ with

$$\sum_n |\langle x^*, x_n\rangle|^p < \infty, \quad \forall x^* \in \mathbf{X}^*.$$

By the closed graph theorem, it is easy to prove that:

Lemma 5. *If a continuous linear operator $T : \mathbf{X} \longrightarrow \mathbf{Y}$ is almost p-summing, then there is a number $K > 0$ such that for all $n = 1, 2, ...$ and for all $x_1, x_2, ..., x_n \in \mathbf{X}$ the following inequality holds true*

$$\Big(\mathbf{E}\Big\|\sum_{k=1}^{n} r_k T x_k\Big\|^2\Big)^{1/2} \le K \sup\Big\{\Big(\sum_{k=1}^{n} |\langle x_k, x^*\rangle|^p\Big)^{1/p}\Big|\ \|x^*\| \le 1\Big\}.$$

Note that it was shown in [11] that:

Proposition 3. *For a Banach space $\mathbf{Y}$ the following statements are equivalent:*
(a) $\mathbf{Y}$ *contains no subspace isomorphic to c_0;*
(b) *A continuous linear operator T from l_2 into $\mathbf{Y}$ is almost 2-summing if and only if there is a number $K > 0$ such that for all $n = 1, 2, ...$ and for all $h_1, h_2, ..., h_n \in l_2$ the following inequality holds true*

$$\mathbf{E}\Big\|\sum_{k=1}^{n} r_k T h_k\Big\|^2 \le K^2 \sup\Big\{\sum_{k=1}^{n} |\langle h_k, h\rangle|^2\Big|\ \|h\| \le 1\Big\}.$$

Remark. The identity of $\mathbf{X}$ is not almost p-summing for any $2 < p < \infty$. In fact, choose $x_k = x$, $k = 1, 2, ...$ with $x \in \mathbf{X}$, $\|x\| = 1$. By the inequality in Lemma 5, for all $n = 1, 2, ...$ we have

$$n^{1/2} \le K.n^{1/p}.$$

This is impossible for $p > 2$.

It was proved in [2, Remark 12.8, p. 235]) that a Banach space $\mathbf{X}$ is finite dimensional if and only if the identity of $\mathbf{X}$ is almost 2-summing. We are now going to study the case $1 \leq p \leq 2$. Consider first the case $p = 1$. It can be checked easily that the identity of the space c_0 is not almost 1-summing. On the other hand, the identity of the space l_1 is almost 1-summing, by the celebrated Schur theorem (see [2, p. 6]). More generally, we have:

Proposition 4. *Let* $\mathbf{X}$ *be a Banach space. The following statements are equivalent:*
(a) $\mathbf{X}$ *contains no subspace isomorphic to* c_0.
(b) *The identity of* $\mathbf{X}$ *is almost 1-summing.*

This proposition is an easy consequence of the following result due to Bessaga-Pelczynsky (see [1] or [2, p. 22]).

Lemma 6. *Let* $\mathbf{X}$ *be a Banach space. The following statements are equivalent:*
(a) $\mathbf{X}$ *contains no subspace isomorphic to* c_0.
(b) *The series* $\sum_n x_n$ *unconditionally converges for any sequence* (x_n) *in* $\mathbf{X}$ *such that*

$$\sum_n |\langle x^*, x_n \rangle| < \infty, \quad \forall x^* \in \mathbf{X}^*.$$

However, for $1 < p \leq 2$ we have:

Theorem 3. *Let* $\mathbf{X}$ *be a Banach space and* p *a real number with* $1 < p \leq 2$. *The following statements are equivalent:*
(a) $\dim \mathbf{X} < \infty$.
(b) *The identity of* $\mathbf{X}$ *is almost* p*-summing.*

To prove this theorem we need the following notion (due to James, see [2, p. 175]).

Definition 5. Let $\mathbf{X}$, $\mathbf{Y}$ be Banach spaces. It is said that $\mathbf{X}$ is finitely representable in $\mathbf{Y}$ if given any $\varepsilon > 0$ and E is a finite dimensional subspace of $\mathbf{X}$ there are a finite dimensional subspace F of $\mathbf{Y}$ and an isomorphism $u : E \longrightarrow F$ such that $\|u\|.\|u^{-1}\| \leq 1 + \varepsilon$.

The following is the heart of the proof of the preceding theorem.

Lemma 7. (Dvorezky Theorem [2, p. 396]). *The space* l_2 *is finitely representable in any Banach space* $\mathbf{X}$ *with* $\dim(\mathbf{X}) = \infty$.

Proof of Theorem 3.
$(a) \Longrightarrow (b)$ is trivial.
$(b) \Longrightarrow (a)$ is a consequence of the following lemmas. First, note that, by Lemma 5, we have:

Lemma 8. *If the identity of a Banach space* $\mathbf{X}$ *is almost p-summing, then there is a number* $K > 0$ *such that for all* $n = 1, 2, ...$ *and for all* $x_1, x_2, ..., x_n \in \mathbf{X}$ *the following inequality holds true*

$$\Big(\mathbf{E}\Big\|\sum_{k=1}^{n} r_k x_k\Big\|^2\Big)^{1/2} \le K \sup\Big\{\Big(\sum_{k=1}^{n} |\langle x_k, x^*\rangle|^p\Big)^{1/p}\Big|\ \|x^*\| \le 1\Big\}. \tag{4.1}$$

By definition, it is obvious to see that:

Lemma 9. *If the inequality* (4.1) *holds true for a Banach space* $\mathbf{Y}$*, so does for any Banach space* $\mathbf{X}$ *which is finitely representable in* $\mathbf{Y}$*.*

The following is the key to our purpose.

Lemma 10. *The inequality* (4.1) *does not hold true for* l_2*, for any* p *with* $1 < p \le 2$*.*

Proof. Let q denote the real number such that $1/p = 1/q + 1/2$. As $1 < p \le 2$, it easy to check that $2 < q \le \infty$ ($q = \infty$ iff $p = 2$). Choose $x_n = \dfrac{1}{\sqrt{n}} e_n$, $n = 1, 2, ...$, where (e_n) is the natural basis in l_2. If the inequality (4.1) does hold true for l_2, then, by the Hölder inequality, for any $n = 1, 2, ...$ we have

$$\begin{aligned}
\sum_{k=1}^{n} \frac{1}{k} &= \Big(\mathbf{E}\Big\|\sum_{k=1}^{n} r_k x_k\Big\|^2\Big)^{1/2} \\
&\le K \sup\Big\{\Big(\sum_{k=1}^{n} |\langle x_k, x^*\rangle|^p\Big)^{1/p}\Big|\ \|x^*\| \le 1\Big\} \\
&= K \sup\Big\{\Big(\sum_{k=1}^{n} |\frac{1}{\sqrt{k}} < e_k, x^* > |^p\Big)^{1/p}\Big|\ \|x^*\| \le 1\Big\} \\
&\le K \sup\Big\{\Big(\sum_{k=1}^{n} (\frac{1}{k})^{q/2}\Big)^{1/q}\Big(\sum_{k=1}^{n} |\langle e_k, x^*\rangle|^2\Big)^{1/2}\Big|\ \|x^*\| \le 1\Big\} \\
&= K\Big(\sum_{k=1}^{n} (\frac{1}{k})^{q/2}\Big)^{1/q}.
\end{aligned}$$

This is impossible for $q > 2$, since

$$\sum_{k=1}^{\infty} \frac{1}{k} = \infty$$

and

$$\sum_{k=1}^{\infty} \frac{1}{k^{q/2}} < \infty.$$

$\square$

By the Dvorezky theorem and the above lemmas we obtain:

Lemma 11. *Let* $\mathbf{X}$ *be a Banach space. If* $\dim \mathbf{X} = \infty$*, then the inequality* (4.1) *does not hold true for* $\mathbf{X}$*, for all* p *with* $1 < p \leq 2$*.*

The proof of Theorem 3 is complete.

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