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LOCALLY LIPSCHITZ SET-VALUED MAPS ON TOPOLOGICAL VECTOR SPACES AND SURJECTIVITY THEOREMS

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Introduction

A subject of great importance in optimization theory is the study of set-valued maps and their properties. Being intermediate between continuity and differentiability, the Lipschitz properties take on a special significance. In Section 1, a definition of Lipschitz set-valued maps in topological vector spaces is given. In Section 2, a surjectivity theorem is proved for Lipschitz approachable set-valued maps defined on a locally convex space (i.e. for those maps which can be Lipschitz approximated by a continuous map having a positive constant of surjection). As consequences of the theorem we obtain again the known results on surjectivity of M. Fabian and D. Preiss [3], P.C. Duong and H. Tuy [2] in Banach spaces.

1. Locally Lipschitz set-valued maps

Let X, Y be two real topological vector spaces and $F : X \to 2^Y$ a set-valued map. We denote

$$CL(Y) = \{A \subseteq Y;\ A \quad \text{closed}\},$$

$$\text{Dom } F = \{x \in X;\ F(x) \neq \emptyset\},$$

$$\text{Graph } F = \{(x, y) \in X \times Y;\ y \in F(x),\ x \in \text{Dom } F\}.$$

Let $z_0 = (x_0, y_0) \in \text{Graph } F$. The following maps are considered:

$$K : X \to 2^Y \quad \text{u.s.c at } 0 \quad \text{and } K(0) = \{0\},$$

$$r :]0,1] \times X \times X \to Y \quad \text{satisfying}$$

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$$\begin{cases} \lim\limits_{t\downarrow 0, x\to x_0} r(t,x,v) &= 0 \quad \text{for each } v \in X \text{ and} \\ \lim\limits_{\substack{t\downarrow 0 \\ x\to x_0, v\to 0}} r(t,x,v) &= 0. \end{cases} \tag{1.1}$$

DEFINITION 1.1. A point-valued map $f : X \to Y$ is called a selection of F at z_0 if $f(x_0) = y_0$ and $f(x) \in F(x)$ for all x sufficiently close to x_0. Such a selection will be denoted by $f_{x_0 y_0}$.

DEFINITION 1.2. We say that F is Lipschitz around x_0 if there exists a neighborhood U of x_0, a neighborhood O of the origin and a number $\eta > 0$ such that for any $x \in U$, $z = (x,y) \in \text{Graph } F$, we can find a selection f_{xy} of F at z satisfying

$$\frac{1}{t}[f_{xy}(x+tv) - y] \in K(v) + r(t,x,v)$$
$$\forall t \in]0,\eta], \ \forall v \in O.$$

DEFINITION 1.3. A point-valued map $g : X \to Y$ is said to be in a Lipschitz proximity of F around x_0 if the set-valued map G defined by $G(x) = F(x) - g(x)$ is Lipschitz around x_0.

Assume that U is a closed circled neighborhood of the origin. The gauge of U is a real function $\varrho_U(x) = \inf\{t > 0;\ x \in tU\}$ for all $x \in X$.

As we know, $\varrho_U(\cdot)$ is positively homogeneous and continuous at the origin with $\varrho_U(0) = 0$. Moreover, $x \in U$ if and only if $\varrho_U(x) \leq 1$. Set

$$d_U(x,B) = \inf\{\varrho_U(x-y);\ y \in B\},$$
$$e_U(A,B) = \sup\{d_U(x,B);\ x \in B\},$$
$$H_U(A,B) = \max\{e(A,B),\ e(B,A)\}.$$

If X is a norm space, the gauge of the unit ball is the norm $||\cdot||$ and

$$d(x,B) = \inf\{||x-y||;\ y \in B\},$$
$$e(A,B) = \sup\{d(x,B);\ x \in B\},$$
$$H(A,B) = \max\{e(A,B),\ e(B,A)\}.$$

PROPOSITION 1.4. *If F is Lipschitz around x, then F is Lipschitz around x in the sense that for any closed circled neighborhood W of the origin in Y, there exist a closed circled neighborhood U of the origin in X and a neighborhood V of x such that*

$$H_W((F(x_1), F(x_2)) \leq \varrho_U(x_2 - x_1), \ \forall x_1, x_2 \in V.$$

PROOF. Assume that F is Lipschitz around x_0, then there exist a neighborhood U_0 of x_0, a neighborhood O of the origin and a number $\eta > 0$ such that $\forall z = (x, y), x \in U_0, \ y \in F(x)$, we can find a selection f_{xy} of F satisfying :

$$\frac{1}{t}[f_{xy}(x + tv) - y] \in K(v) + r(t, x, v) \qquad (1.4.1)$$

$$\forall t \in]0, \eta], \ \forall v \in O.$$

Let W be a closed circled neighborhood of the origin in Y. Choose W_1, a neighborhood of the origin such that

$$W_1 + W_1 \subseteq W.$$

Since K is u.s.c at 0, $K(0) = \{0\}$ and $\lim\limits_{\substack{v \to 0 \\ t \downarrow 0, x \to x_0}} r(t, x, v) = 0$, we can choose then a closed circled neighborhood U of the origin in X and a number $\gamma > 0$ satisfying

$$K(v) \subseteq W_1 \quad \forall x \in U \quad \text{and} \quad r(t, x, v) \subseteq W_1 \quad \forall t \in]0, \gamma], \ \forall x \in x_0 + U, \ \forall v \in U.$$

(In fact, we can take $\gamma \leq \eta$ and $U \subseteq O$). Hence, for any $t \in]0, \gamma], \ v \in U$, we have in view of (1.4.1)

$$\frac{1}{t}[f(x + tv) - y] \in W_1 + W_1 \subseteq W,$$

or equivalently

$$f_{xy}(x + \omega) - y \in tW, \ \forall t \in]0, \gamma], \ \forall \omega \in tU. \qquad (1.4.2)$$

Let $V = x_0 + \frac{1}{4}\gamma U$ be a neighborhood of x_0. We have to show that

$$H_W(F(x_1), F(x_2)) \leq \varrho_U(x_2 - x_1), \ \forall x_1, x_2 \in V.$$

Indeed, let $x_1, x_2 \in V$. We have $x_2 - x_1 \in \frac{1}{2}\gamma U$ and $\varrho_U(x_2 - x_1) \leq \frac{1}{2}\gamma < \gamma$. Let α be a positive number satisfying $\varrho_U(x_2 - x_1) \leq \alpha < \gamma$. Then

$$x_2 - x_1 \in \alpha U. \tag{1.4.3}$$

Now, for every $y_1 \in F(x_1)$, there exists a selection $f_{x_1 y_1}$ of F such that

$$f_{x_1 y_1}(x_1 + \omega) - y_1 \in tW, \ \forall t \in]0, \gamma], \ \forall \omega \in tU.$$

Let $\omega = x_2 - x_1$. Then we get by (1.4.3) :

$$f_{x_1 y_1}(x_1 + \omega) - y_1 = f_{x_1 y_1}(x_2) - y_1 \in \alpha W.$$

Since $y_2 = f_{x_1 y_1}(x_2) \in F(x_2)$ and $\varrho_W(y_2 - y_1) \leq \alpha$, we obtain

$$d_W(y_1, F(x_2)) \leq \alpha.$$

Moreover, this inequality holds for every $y_1 \in F(x_1)$, i.e.

$$e_W(F(x_1), F(x_2)) \leq \alpha.$$

By symmetry we also have

$$e_W(F(x_2), F(x_1)) \leq \alpha$$

Consequently,

$$H_W(F(x_1), F(x_2)) \leq \alpha.$$

Since α can be arbitrarily chosen in the interval $[\varrho_U(x_2 - x_1), \gamma]$ we get

$$H_W(F(x_1), F(x_2)) \leq \varrho_U(x_2 - x_1).$$

As a consequence of Proposition 1.4, we have the following corollary:

COROLLARY 1.5. *If F is Lipschitz around x_0, then F is continuous at x_0.*

REMARK.

1) If X and Y are norm spaces, Definition 1.2 reduces to the usual definition of the Lipschitz continuity, i.e. there exist a neighborhood U of x_0 and a number

$L > 0$ such that

$$H(F(x), F(y)) \leq L||x - y||, \ \forall x, y \in U.$$

2) In the case F is point-valued and $K(v)$ is compact (or bounded) for every $v \in X$, the Lipschitz property in the sense of Definition 1.2 means the strictly compactly Lipschitz (or strictly boundedly Lipschitz) property in Thibault's sense (see [7]).

2. Surjectivity theorems

Let X be a locally convex space and Y a norm space. We recall that a map F from X to Y is surjective at a point x_0 if for every neighborhood U of x_0, the set $F(U)$ is a neighborhood of $F(x_0)$.

Let U be a closed bounded circled neighborhood of the origin in X and $g : X \to Y$ be a point-valued map. Then the surjectivity of g can be characterized by a quantity called the constant of surjection.

DEFINITION 2.1. The function

$$t \to M_U(g, x_0, t) = \frac{1}{t} \sup\{s > 0;\ g(x_0) + sB \subseteq \overline{g(x_0 + tU)}\}$$

is called the modulus of surjection of g at x_0 with respect to U.

The quantity

$$C_U(g, x_0) = \lim_{\substack{x \to x_0 \\ t \downarrow 0}} M_U(g, x, t)$$

is called the constant of surjection of g at x_0 with respect to U.

If X is a norm space, we denote by $C(g, x_0)$ the constant of surjection of g at x_0 with respective to the unit ball and

$$C(g, x_0) = \lim_{\substack{x \to x_0 \\ t \downarrow 0}} \frac{1}{t} \sup\{s > 0;\ B_Y(g(x), s) \subseteq \overline{g(B_X(x, t))}\}.$$

LEMMA 2.2. *Let $\beta > 0$ and g a linear point-valued map from a norm space X to a norm space Y. Assume that for each $y \in Y$ there exists $x \in X$ satisfying*

$$g(x) = y \qquad \text{and} \qquad \beta||x|| \leq ||y||.$$

Then $C(g, x_0) \geq \beta$ for any $x_0 \in X$.

PROOF. Since g is linear, $y = g(x)$ and $\beta||x|| \leq ||y||$, we have

$$t\beta B_Y \subseteq g(tB_X) \quad \forall t > 0.$$

Again by the linearity of g we get

$$g(x) + t\beta B_Y \subseteq g(x + tB_X), \ \forall x \in X, \ \forall t > 0.$$

This means that

$$M(g, t, x) = \frac{1}{t} \sup\{s > 0; g(x) + sB_Y \subseteq \overline{g(x + tB_X)}\} \geq \beta.$$

It follows then

$$C(g, x_0) \geq \beta, \quad \forall x_0 \in X.$$

Let G be a convex process from X to Y, i.e., Graph G is a convex cone. The inverse of G is a convex set-valued map G^{-1} defined by $G^{-1}(y) = \{x; y \in G(x)\}$. The norm $||G^{-1}||$ of G^{-1} is defined as the smallest number $\gamma \geq 0$ such that for every y in the range of G there is a point $x \in G^{-1}(y)$ satisfying $||x|| \leq \gamma||y||$.

LEMMA 2.3. *Let X, Y be norm spaces, $G : X \to 2^Y$ a closed convex set-valued and $P : X \times Y \to Y$ defined by $P(x, y) = y$. Denote by $P_{|\text{Graph}}$ the restriction of P on Graph G. If $||G^{-1}|| > 0$, then*

$$C(P_{|\text{Graph}G}, (x, y)) \geq ||G^{-1}||^{-1}, \ \forall x \in X, \ y \in G(x).$$

PROOF. Let $Z = X \times Y$ be equipped with $||(x, y)|| = \max\{||x||, \ ||G^{-1}|| \, ||y||\}$. Then for every y in the range of G, there is $(x, y) \in X \times Z \cap \text{Graph } G$ satisfying

$$P(x, y) = y \quad \text{and} \quad \frac{1}{||G^{-1}||}||(x, y)|| \leq ||y||.$$

Hence Lemma 2.3 follows from Lemma 2.2.

Now let g be a point-valued map from a locally convex space X into a norm space Y. Assume that g is in a Lipschitz proximity of F around x_0, then by Proposition 1.4 we have

$$H(F(x_1) - g(x_1), F(x_2) - g(x_2)) \leq \varrho_{\overline{U}}(x_2 - x_1) \quad \forall x_1, x_2 \in V$$

for some neighborhood $\overline{U}$ of the origin and some neighborhood V of x_0.

Since U is bounded, there exists a constant $L > 0$ such that $LU \supseteq \overline{U}$ and hence $L\varrho_U(x) \geq \varrho_{\overline{U}}(x)$, $\forall x \in X$. We get

$$H(F(x_1) - g(x_1), F(x_2) - g(x_2)) \leq L\varrho_U(x_2 - x_1) \quad \forall x_1, x_2 \in V. \qquad (*)$$

DEFINITION 2.4. We say that g is a (U, L)-Lipschitz approximating of F around x_0 if (*) holds.

THEOREM 2.5. *Let X be a complete locally convex space, Y a norm space, $F : X \to CL(X)$ a closed set-valued map and $g : X \to Y$ a continuous point-valued map. Assume that g is a (U, L) - Lipschitz approximating of F around x_0 and $C_U(g, x_0) > L > 0$. Then F is surjective at x_0.*

PROOF. Since $L < C_U(g, x_0)$, we can choose $\theta, \epsilon \in]0, 1[$ such that $L + \theta < C_U(g, x_0) - \epsilon$. Put $\gamma = C_U(g, x_0) - \epsilon$. Then there exist a number $b > 0$ and a neighborhood M of the origin satisfying

$$M_U(g, x, t) \geq \gamma, \ \forall t \in]0, b], \ \forall x \in x_0 + M,$$

i.e.

$$\frac{1}{t} \sup\{s > 0; g(x) + sB \subseteq \overline{g(x + tU)}\} \geq \gamma$$

$$\sup\{s > 0; g(x) + sB \subseteq \overline{g(x + tU)}\} \geq \gamma t, \ \forall t \in]0, b], \forall x \in x_0 + M.$$

This means that

$$g(x) + \gamma tB \subseteq \overline{g(x + tU)}, \ \forall t \in]0, b], \ \forall x \in x_0 + M. \qquad (2.5.1)$$

On the other hand, since g is a (UL)-Lipschitz approximating of F around x_0, there is a neighborhood N of the origin such that:

$$H(F(x) - g(x), F(z) - g(z)) \leq L\varrho_U(z - x),\ \forall x, z \in x_0 + N. \tag{2.5.2}$$

Let $V = M \cap N$ be a neighborhood of the origin and take $t_0 \in]0, b]$ such that $t_0 U \subseteq V$. Then we have $tU \subseteq V$ for all $t \in]0, t_0]$.

Put $s = (L+\theta)\gamma^{-1} < 1$. We shall prove that for any $y \in F(x_0) + \gamma(1-s)t_0 B$, there exists an element $\bar{x} \in x_0 + t_0 U$ such that $y \in F(\bar{x})$.

First, we construct two sequences $\{y_n\}$ and $\{z_n\}$ in Y satisfying

1) $y_n \in F(x_n) : x_n \in x_{n-1} + s^{n-1}(1-s)t_0 U$.

2) $z_n = g(x_n) - y_n + y,\ ||y_n - y|| = ||z_n - g(x_n)|| \leq \theta\varrho_U(x_{n+1} - x_n)$.

Take $y_0 \in F(x_0)$ such that $||y_0 - y|| \leq \gamma(1-s)t_0$ and $z_0 = g(x_0) - y_0 + y$. Then we have

$$||z_0 - g(x_0)|| = ||y - y_0|| \leq \gamma(1-s)t_0.$$

Hence $z_0 \in g(x_0) + \gamma(1-s)t_0 B$. It follows from (2.5.1) :

$$z_0 \in \overline{g(x_0 + (1-s)t_0 U)}. \tag{2.5.3}$$

Let $a_0 = ||z_0 - g(x_0)|| = ||y - y_0||$. If $a_0 = 0$, then $y = y_0$ and so $\bar{x} = x_0$ satisfying $y \in F(\bar{x})$. If $a_0 > 0$, by the continuity of g, there is a neighborhood U_0 of the origin such that

$$||g(x) - g(x')|| \leq \frac{a_0}{2},\ \forall x, x' \in x_0 + U.$$

Take $\delta_0 > 0 : \delta_0 U \subseteq U_0$. Then we have

$$||g(x) - g(x')|| \leq \frac{a_0}{2},\ \forall x, x' \in x_0 + \delta_0 U.$$

In view of (2.5.3) we can choose $x_1 \in x_0 + (1-s)t_0 U$ satisfying :

$$||z_0 - g(x_1)|| \leq \theta \min\{\frac{a_0}{2}, \delta_0\}.$$

Evidently, we have $x_1 \notin x_0+\delta_0 U$, i.e. $\varrho_U(x_1-x_0) > \delta_0$. In the contrary case we have $||g(x_1)-g(x_0)|| < \frac{a_0}{2}$ and since $||z_0-g(x_0)|| < \frac{a_0}{2}$ we get $||z_0-g(x_0)|| < a_0$, a contradiction. Thus we have

$$||z_0 - g(x_1)|| \leq \theta\delta_0 \leq \theta\varrho_U(x_1 - x_0).$$

Since $x_0, x_1 \in x_0 + N$, by (2.5.2) we obtain

$$H(F(x_1) - g(x_1),\ F(x_0) - g(x_0)) \leq L\varrho_U(x_1 - x_0).$$

Now choose $y_1 \in F(x_1)$ such that

$$||y_1 - g(x_1) - y_0 + g(x_0)|| \leq L\varrho_U(x_1 - x_0).$$

Put $z_1 = g(x_1) - y_1 + y$. Then

$$\begin{aligned}
&||z_1 - g(x_1)|| \leq ||y_1 - g(x_1) - y_0 + g(x_0)|| + ||g(x_0) - y_0 + y - g(x_1)|| \\
&||z_1 - g(x_1)|| \leq L\varrho_U(x_1 - x_0) + ||z_0 - g(x_1)|| \\
&||z_1 - g(x_1)|| \leq L\varrho_U(x_1 - x_0) + \theta\varrho_U(x_1 - x_0) \leq (L+\theta)(1-s)t_0 \\
&||z_1 - g(x_1)|| \leq \gamma(1-s)st_0,
\end{aligned}$$

i.e. $z_1 \in g(x_1) + \gamma(1-s)st_0B$. In view of (2.5.1) we get

$$z_1 \in \overline{g(x_1 + s(1-s)t_0U)}. \tag{2.5.4}$$

Put $a_1 = ||z_1 - g(x_1)||$. If $a_1 = 0$ we take $\bar{x} = x_1$ and then $y = y_1 \in F(\bar{x})$. If $a_1 > 0$, we choose $\delta_1 > 0$ and U_1 a neighborhood of the origin such that

$$||g(x) - g(x')|| \leq \frac{a_1}{2},\ \forall x, x' \in x_1 + \delta_1 U \subseteq U_1.$$

Let $x_2 \in x_1 + (1-s)st_0U$ satisfying

$$||z_1 - g(x_2)|| \leq \theta \min\{\frac{a_1}{2}, \delta_1\} \quad \text{(in view of (2.5.4))}.$$

We then have

$$\varrho_U(x_2 - x_1) \leq (1-s)st_0.$$

Again we choose $y_2 \in F(x_2)$ such that (in view of (2.5.1)) :

$$||y_2 - g(x_2) - y_1 + g(x_1)|| \leq L\varrho_U(x_2 - x_1).$$

Let $z_2 = g(x_2) - y_2 + y$. Then

$$||z_2 - g(x_2)|| \leq ||y_2 - g(x_2) - y_1 + g(x_1)|| + ||z_1 - g(x_2)||$$
$$||z_2 - g(x_2)|| \leq (L + \theta)\varrho_U(x_2 - x_1) \leq \gamma(1-s)s^2 t_0 U,$$

i.e.

$$z_2 \in g(x_2) + \gamma(1-s)s^2 t_0 B \subseteq \overline{g(x_2 + (1-s)s^2 tU)}.$$

Continuing this procedure, we obtain two sequences $\{y_n\}$ and $\{z_n\}$ in Y which satisfy conditions 1) and 2). Hence, $y_n \underset{n\to\infty}{\longrightarrow} y$ in view of 2).

Now we show that $\{x_n\}$ is a Cauchy sequence, i.e., for any neighborhood $\overline{V}$ of the origin in X (we can obviously assume that $\overline{V}$ is closed, circled) there is an integer k such that $x_m - x_n \in \overline{V}$, $\forall m, n \geq k$.

Since $s < 1$, we can find k such that $s^k t_0 U \subset \overline{V}$. Let $m > n > k$. From 1) we get

$$x_m - x_{m-1} \in (1-s)s^{m-1} t_0 U \subseteq (1-s)s^{m-k-1}\overline{V},$$
$$x_{n+1} - x_n \in (1-s)s^n t_0 U \subseteq (1-s)s^{n-k}\overline{V}.$$

Put $v = x_m - x_n = x_m - x_{m-1} + x_{m-1} - x_{m-2} + \cdots + x_{n+1} - x_n$. Then

$$v = (1-s)s^{m-k-1} v_{m-k-1} + \cdots + (1-s)s^{n-k} v_n$$

where

$$v_i \in \overline{V} \quad (\text{or} \quad \varrho_{\overline{V}}(v_i) \leq 1), \quad \forall i \overline{n-k, m-k-1}.$$

So

$$\varrho_{\overline{V}}(v) \leq (1-s)s^{m-k-1}\varrho_{\overline{V}}(v_{m-k-1}) + \cdots + (1-s)s^{n-k}\varrho_{\overline{V}}(v_n),$$
$$\varrho_{\overline{V}}(v) \leq (1-s)s^{n-k}(s^{m-n-1} + \cdots + 1) \leq s^{n-k}.$$

This leads to

$$\varrho_{\overline{V}}(v) \leq 1, \quad \text{i.e,} \quad v = x_m - x_n \in \overline{V}.$$

By the completeness of X, the sequence $\{x_n\}$ converges to a point $\bar{x} \in x_0 + t_0 U$. Since F is a closed map, $y \in F(\bar{x})$ and the theorem is proved.

COROLLARY 2.6. *Let X, Y be Banach spaces, $F : X \to CL(Y)$ a closed set-valued map and $g \in L(X, Y)$ satisfying the condition that for each $y \in Y$, there is $x \in X$ such that $g(x) = y$ and $||y|| \geq \beta||x||$. Assume that g is a (U, L)-Lipschitz approximating of F around x_0 and $\beta > L > 0$. Then F is surjective at x_0.*

PROOF. Applying Lemma 2.2, we get $C(g, x_0) \geq \beta > L$, hence the result follows.

REMARK. By letting F (in Corollary 2.6) be point-valued, we obtain a result of M. Fabian and D. Preiss in [3].

Now, let $f, g : X \to Y$ be a point-valued map and M a closed convex cone in Y. We define $F, G : X \to 2^Y$ as follows:

$$F(x) = f(x) - M \quad \text{and} \quad G(x) = g(x) - M$$

We obtain the following corollary which is a result of P.C. Duong and H. Tuy in [2] (cf. S.M. Robinson [6]).

COROLLARY 2.7. *Assume that f, g are continuous and satisfy the following conditions:*

1) G is a surjective closed convex map

2) $0 \leq K||G^{-1}|| < 1$

3) $||f(x) - f(x') - g(x) + g(x')|| \leq K||x - x'||$, $\forall x, x' \in B(0, r)$.

Then F is surjective at x_0.

PROOF. Let $Z = X \times Y$ be equipped with $||(x, y)|| = \max\{||x||, ||G^{-1}|| \, ||y||\}$, $P : X \times Y \to Y$ defined by $P(x, y) = y$ and ℓ : Graph $G \to Y$ defined by

$$\ell(x, y) = f(x) - g(x) + y.$$

By Lemma 2.3, $C(P_{|\text{Graph } G}(x_0, f(x_0)) \geq ||G^{-1}||$. On the other hand, 3) means that P is a (B, K)-Lipschitz approximating of ℓ, so that by Theorem

2.5:

$$f(x_0) = y_0 = \ell(x_0, y_0) \in \text{Int } (P(B_r(x_0, y_0) \cap \text{Graph } G)) \text{ for some } \quad r > 0,$$

i.e.

$$y_0 \in \text{Int}(\ell(U \times T \cap \text{Graph } G) \text{ for every neighborhood of } x_0.$$

But

$$\ell(U \times Y \cap \text{Graph } G) = \{f(x) - g(x) + y; x \in U, \ y \in g(x) - M\}$$
$$= \{f(x) - M; \ x \in U\} = F(U)$$

This means that F is surjective at x_0.

References

[1] F.H. Clark, "Optimization and nonsmooth analysis," Wiley-Interscience publication, 1983.

[2] P.C. Duong and H. Tuy, *Stability, surjectivity and local invertibility of nondifferentiable mappings*, Acta Math. Vietnam. **3** (1978), 89–105.

[3] M. Fabian and D. Preiss, *A generalization of the interior mapping theorem of Clarke and Pourciau*, Com. Math. Univ. Carolinae **28** (1987), 311–324.

[4] L.V. Hot, *On the open mapping principle and convex multi-valued mappings*, Acta Univer. Carolinae Math. et Phyz. **26** (1985), 53–59.

[5] A.D. Ioffe, *Nonsmooth analysis: Differential calculus of nondifferentiable mappings*, Trans. Amer. Math. Soc. **206** (1981), 11–55.

[6] S.M. Robinson, *Regularity for convex multivalued functions*, Math. of Oper. Resear. 1 (1976), 130-143.

[7] L. Thibault, Sur les fonctions compactement Lipschitziennes et leurs applications, Univ. du. Languedoc 1980.

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