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OKA-WEIL THEOREM AND PLURISUBHARMONIC FUNCTIONS OF UNIFORM TYPE

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Introduction

The classical Oka-Weil theorem is well-known : Let K be a compact polynomially convex subset of the space C^n. Then every holomorphic function in a neighbourhood of K can be uniformly approximated by a sequence of polynomials on K. This theorem has been generalized in various directions by several authors. Noverzaz [6] proved an analogous theorem in Banach spaces with bounded approximation property. He considered also the approximation of continuous plurisubharmonic functions on a pseudoconvex domain in Banach spaces by special function of the form

$$\sup_{i=1,2,\dots,p} a_i \log |f_i(z)|,$$

where $f_i \in H(U)$ and $a_i > 0,\ i = 1,2,...,p$. See also the works of Matyszczyk [2] and Mujica [5] for the case of Frechet spaces.

This paper establishes a version of the classical Oka-Weil theorem for sequential approximation of plurisubharmonic functions defined on pseudoconvex domains in Fréchet spaces with a Schauder basis.

Using this result we characterize the property $\overline{\overline{\Omega}}$ (introduced by Vogt [9]) of nuclear Fréchet spaces with a Schauder basis by the uniformity of plurisubharmonic functions.

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1. Approximation of continuous plurisubharmonic functions in Fréchet spaces.

Recall that an open subset Ω of a Fréchet space E is said to have the property P if $\widehat{K}_{PS(E)}$ is relatively compact in Ω for every compact subset K in Ω, where

$$\widehat{K}_{PS(E)} = \{x \in E : f(x) \leq \sup_{x \in K} f(x),\ f \in PS(E)\},$$

and $PS(E)$ is the space of all plurisubharmonic function on E.

In this section we give a necessary and sufficient condition for the approximation of continuous, plurisubharmonic functions on every open set Ω having the property P by a sequence of finite supremum of functions a log $|f|$ with $a > 0$ and f in $H(U)$.

THEOREM 1.1. *Let E be a Fréchet space having a Schauder basis. If for every open subset Ω of E with the property P and for every continuous, plurisubhamornic function f on Ω, there exists a sequence of functions $\{f_n\}$ uniformly convergent to f on each compact subset of Ω :*

$$f_n(z) = \max_{j=1,\dots,m_n} a_j^n \log |f_j^n(z)|,\ a_j^n > 0,$$

where f_j^n are holomorphic functions on Ω, then E has a continuous norm.

PROOF: Let $\{e_k\}$ be a Schauder basis of E. First, as in [8], we prove that there exists a sequence $\{\lambda_n\}$ such that $\lambda_{n_k} e_{n_k} \not\to 0$ for every subsequence $\{\lambda_{n_k}\} \subset \{\lambda_n\}$. Consider a polynomially convex subset D in $\mathbb{C}$ consisting of infinite many convex components :

$$D = \bigcup_{j=1}^{\infty} D_j,\ 0 \in D.$$

Put

$$M = \overline{\operatorname{span}\{e_j\}_{j \geq 2}},$$

$$\Omega = \bigcup_{j=1}^{\infty} D_j e_1 \oplus M$$

Then Ω has the property P. Define a plurisubharmonic function on Ω :

$$f(z) = |e_j^*(z)| = |z_j| \quad \text{for} \quad z \in D_j e_1 \oplus M.$$

By the assumption, there exists a sequence of functions $\{f_n\}$:

$$f_n(z) = \max_{j \leq m_n} a_j^n \log |f_j^n(z)|$$

with $a_j^n > 0$, $n = 1, 2, ...$, such that the sequence $\{f_{n|D_j e_1} \oplus \mathbb{C}e_j\}_{n \in \mathbb{N}}$ uniformly converges on each compact subset of $D_j e_1 + \mathbb{C}e_j$ to $f(z) = |z_j|$, here $z = z_1 e_1 + z_j e_j$. This means that for n sufficiently large, f_n is dependent on the variable z_j. It follows that one of the functions f_i^n, $i = 1, 2, ..., m_n$, is dependent on z_j. Hence there exists $z_1^j < \frac{1}{j}$ such that $f_{i_j}^{n_j}(z_1^j, z_j)$ is unbounded on $z_1^j e_1 \oplus Ce_j$ for some index n_j and i_j. This implies the existence of $\lambda_j \in \mathbb{C}$ such that

$$| f_{n_j}(z_1^j e_1 + \lambda_j e_j) | > j$$

we shall show that $\{\lambda_j\}$ is the desired sequence. Assume for the contrary that $\lambda_{j_k} e_{j_k} \to 0$ for a subsequence $\{\lambda_{j_k}\} \subset \{\lambda_j\}$. Put

$$K = \{z_1^{j_k} e_1 + \lambda_{j_k} e_{j_k},\ k = 1, 2, ...\} \cup \{0\}.$$

Then K is a compact subset of $D_{e_\ell} \oplus M$ for some index ℓ such that $0 \in D_\ell$. We have

$$\begin{aligned} &|f_{n_{j_k}}(z_1^{j_k} e_1 + \lambda_{j_k} e_{j_k}) - |e_\ell^*(z_1^{j_k} e_1 + \lambda_{j_k} e_{j_k})|| = \\ &|f_{n_{j_k}}(z_1^{j_k} e_1 + \lambda_{j_k} e_{j_k})| \geq j_k \quad \text{for} \quad k > \ell. \end{aligned}$$

This contradicts the fact that $\{f_n\}$ converges uniformly on K. Since

$$\lim_{j \to \infty} (e_j^*(z)/\lambda_j).\lambda_j e_j = \lim_{j \to \infty} e_j^*(z) e_j = 0$$

for every $z \in E$, we have $\lim_{j \to \infty} e_j^*(z)/\lambda_j = 0$. Hence $||z|| = \sup_{j \geq 1} |\frac{e_j^*(z)}{\lambda_j}|$ defines a continuous norm on E.

THEOREM 1.2. *Let E be a Fréchet space with a Schauder basis and a continuous norm. If $\Omega \subset E$ is an open subset with the property P, then every continuous plurisubharmonic function f on Ω can be uniformly approximated on every compact subset of Ω by a sequence of functions :*

$$f_n(z) = \max_{j \leq m_n} a_j^n \log|f_j^n(z)|, \; a_j^n > 0,$$

where f_j^n are holomorphic on Ω.

PROOF: Set $A_n(x) = \sum_{j=1}^{n} e_j^*(x)e_j$, where $\{e_j\}$ is a Schauder basis of E. Then the sequence $\{A_n\}$ uniformly convergences on each compact subset of E to the identity operator. Since Ω is an open subset of the Fréchet space E, we can write $\Omega = \bigcup_{n \in N} F_n$, where F_n is closed, $F_n \subset F_{n+1}$ and Int $F_n \neq \emptyset$, $n \in N$. Let

$$\Omega_j = \{x \in \Omega : ||x|| < j\},$$
$$K_j = \overline{F_j \cap \Omega_j \cap A_j(E)}$$

and $||\cdot||$ be a continuous norm on E. Then

$$K_j \subset F_j \cap A_j(E) \subset \Omega \cap A_j(E),$$

and K_j is compact in $\Omega \cap A_j(E)$ for $j \in N$. Consider the restriction $f|_{\Omega \cap A_j(E)}$. By [6], there exist $f_k^j \in H(\Omega \cap A_j(E))$, $k = 1, 2, ...,$ such that

$$||f_j - f||_{K_j} < \frac{1}{j},$$

where $f_j(z) = \max_{k \leq m_j} a_k^j \log|f_k^j(z)|$. Since $\Omega \cap A_j(E)$ has the property P with respect to $A_j(E)$, the functions f_k^j can be replaced by holomorphic functions on $A_j(E)$ [4, Theorem 2.4]. We shall show that $\{f_j \circ A_j\}$ converges uniformly on each compact subset K in Ω. Choose n_0 such that $K \subset$ Int F_{n_0}. Then there exists a neighbourhood of zero in E such that

$$K + V \subset K + \overline{V} \subset \text{Int } F_{n_0}, \tag{1.1}$$

$$A_j(K) \subset K + V \quad \text{for all} \quad j \geq j_0. \tag{1.2}$$

From (1.1) and (1.2) we obtain

$$A_j(K) \subset F_{n_0} \subset F_j \quad \text{for all} \quad j \geq j_1 = \max(j_0, n_0). \tag{1.3}$$

Since $\bigcup_{j \geq j_1} A_j(K)$ is relatively compact, $\bigcup_{j \geq j_1} A_j(K) \subset \Omega_{j_2}$ for some $j_2 \geq j_1$. Hence

$$A_j(K) \subset \Omega_j \quad \text{for all} \quad j \geq j_2 \tag{1.4}$$

From (1.3) and (1.4) we have

$$A_j(K) \subset \Omega_j \cap F_j \cap A_j(E) \subset K_j \quad \text{for all} \quad j \geq j_2.$$

Thus

$$\begin{aligned} &\|f_j A_j - f\|_K \leq \|f_j A_j - f A_j\|_K + \|f A_j - f\|_K \\ &= \|f_j - f\|_{A_j K} + \|f A_j - f\|_K \leq \|f_j - f\|_{K_j} + \|f A_j - f\|_K \\ &\frac{1}{j} + \|f A_j - f\|_K \to 0 \quad (\text{as} \quad j \to \infty). \end{aligned}$$

Combining Theorem 1.1 and Theorem 1.2 we get the following

THEOREM 1.3. *Let E be a Fréchet space having a Schauder basis. Then E has a continuous norm if and only if for every open subset Ω of E with the property P and for every continuous plurisubharmonic function f on Ω, there exists a sequence of function $\{f_n\}_n$ uniformly convergent to f on each compact subset of Ω :*

$$f_n(z) = \max_{j \leq m_n} a_j^n \log |f_j^n(z)|,$$

where $a_j^n > 0$ and $f_j^n \in H(\Omega)$.

2. Plurisubharmonic function of uniform type

DEFINITION: Let E be a locally convex space and G an open set in $\mathbb{C}^k$. A plurisubharmonic function $\varphi : G \times E \longrightarrow [-\infty, \infty)$ is called uniformly plurisubharmonic (or of uniform type) if there exists a continuous seminorm p on E and a plurisubharmonic function φ on $G \times E_p$ such that

$$\varphi = g.(\mathrm{Id}G \times \Pi_p),$$

where E_p is the completion of the canonical normed space $E/\ker p$ and Π_p is the canonical projection from E on the space E_p.

It should be noted that in the particular case when φ depends only on the second variable $z \in E$, it is considered as a function on E and is uniformly plurisubharmonic in the usual sense.

Recently, Meise and Vogt have investigated the relation between the uniform boundedness of holomorphic functions and the property $\bar{\bar{\Omega}}$ of a nuclear Fréchet space (see [3, Theorem 3.3]). Recall that a locally convex space E is said to have the property $\bar{\bar{\Omega}}$ if for each continuous seminorm p on E, there exists a continuous seminorm q on E such that for each continuous seminorm k on E and each $\varepsilon > 0$ there exists $c > 0$ with

$$||y||_q^{*^{1+\epsilon}} \leq C\ ||y||_k^*\ ||y||_p^{*^{\epsilon}} \quad \text{for all} \quad y \in E'.$$

here $||y||_p^* = \sup\{|y(x)| : p(x) \leq 1\}$. It is well-known that uniformly bounded holomorphic functions are closely connected with uniformly plurisubharmonic functions. Thus, it is natural for us to give a characterization of the property $\bar{\bar{\Omega}}$ by the uniformity of plurisubharmonic functions.

THEOREM 2.1. *Let E be a nuclear Fréchet space with a Schauder basic. Then E has the property $\bar{\bar{\Omega}}$ if and only if every plurisubharmonic function of class C^1 on $G \times E$ is of uniform type, where G is an open bounded absolutely convex subset of $\mathbb{C}^k$.*

PROOF: a) Suppose that E has the property $\overline{\overline{\Omega}}$ and let G be an open, bounded absolutely convex subset of $\mathbb{C}^k$. Let φ be a plurisubharmonic function of the class C^1 on $G \times E$.

First, assume that E has a continuous norm. Since $G \times E$ has the property P, by Theorem 1.2 there exist functions $f_j^n \in H(\Omega \times E)$, $n = 1, 2, \ldots$, and $j \leq m_n$, such that

$$\varphi(x, z) = \lim_{n \to \infty} f_n(x, z),$$

where $f_n(x, z) = \max_{j \leq m_n} a_j^n \log |f_j^n(x, z)|$. Moreover, the sequence $\{f_n\}$ uniformly converges to φ on each compact subset of $G \times E$. There exists a balanced convex neighbourhood U of zero in E such that

$$\sup_{\substack{x \in \rho\overline{G} \\ z \in U}} \max_{j \leq m_n} a_j^n \log |f_j^n(x, z)| = M_\rho < \infty$$

for every $0 < \rho < 1$. Hence

$$|f_j^n(x, z)| \leq e^{M_\rho / a_j^n}$$

for all $x \in \rho\overline{G}$, $z \in U$, $n > 0$, $j \leq m_n$.

As in [3], we can find a balanced convex neighbourhood $V \subset U$ and holomorphic functions $\tilde{f}_j^n$ on $G \times E_V$ such that

$$f_j^n(x, z) = \tilde{f}_j^n(x, \Pi_V(z)),$$

and

$$\sup\{|\tilde{f}_j^n(x, z)| : (x, z) \in \rho\overline{G} \times E_V, \ ||z|| \leq r\} \leq C_r e^{M_\rho / a_j^n}$$

for some constant C_r dependent only on r, here E_V is the completion of $E/\text{Ker}\, P_V$ considered as a norm space by the Minkowski functional P_V associated with V and $\Pi_V : E \to E_V$ is the canonical projection. We may assume that $0 < a_j < 1$ for all n and $j \leq m_n$. Then

$$a_j^n \log |\tilde{f}_j^n(x, z)| \leq a_j^n \log C_r + M_\rho \leq \log C_r + M_\rho,$$

for all $n > 1$, $j \leq m_n$ and $(x, z) \in \rho\overline{G} \times E_V$, $||z|| \leq r$. Define a function $\tilde{\varphi}$ on $G \times E_V$ by

$$\tilde{\varphi}(x, z) = \lim_{z' \to z} \sup[\lim_{n \to \infty} \max_{j \leq m_n} a_j^n \cdot \log |f_j^n(x, z)|]$$

Then $\tilde{\varphi}$ is plurisubharmonic on $G \times E_V$ [7].

Since the function φ belongs to the class C^1, we find a neighbourhood $G_0 \times W$ of zero in $G \times E$ and a continuous plurisubharmonic function φ' on $G_0 \times W$ such that

$$\varphi(x, z) = \varphi'(x, \Pi(z)).$$

Consider the plurisubharmonic function

$\tilde{\varphi}(IdG \times \Pi_{WV})$ on $G \times E_W$, where

$\Pi_{WV} : E_W \to E_V$ is the canonical projection.

By the continuity of φ' on $G_0 \times W$,

$$\tilde{\varphi}(IdG_0 \times \Pi_{WV}) |_{G_0 W} = \varphi'.$$

Hence

$$\tilde{\varphi}(IdG \times \Pi_{WV} \circ \Pi_V) = \varphi.$$

Thus φ is uniformly plurisubharmonic.

Now we can pass to the general case, where E does not need to have a continuous norm. Since φ belongs to the class C^1 in a neighbourhood of zero in $G \times E$, we can find a continuous seminorm α on E and a neighbourhood of zero in G such that φ and its derivative is bounded on $G_0 \times \{z \in E : \alpha(z) < 1\}$. Without loss of generality, we may assume that

$$\alpha(z) = \sup\{\alpha(\pi_n(z)) : n \in N\},$$

where

$$\pi_n(z) = \sum_{k=1}^{n} e_k^*(z) e_k.$$

Put

$$Z^\alpha = \{n \in N : \alpha(e_n) = 0\},$$
$$E^\alpha = \{z \in E : e_n^*(z) = 0,\ n \in Z^\alpha\}.$$

Then by [5, Lemma 3.1], $E =\mathrm{Ker}\ \alpha + E^\alpha$, and E^α has a Schauder basis and a continuous norm. For every $z \in E$, we write $z = z_1 + z_2,\ z_1 \in \mathrm{Ker}\ \alpha,\ z_2 \in E^\alpha$. We shall show that the function φ is not dependent on the component z_1. Let $z_1 \in \mathrm{Ker}\ \alpha$. For each $z_2 \in E$ with $\alpha(z_2) < 1$, consider the subharmonic function $(x, \lambda) \to \varphi(x, z_1 + \lambda z_2)$. Since φ belongs to the class C^1, for λ sufficient small and $x \in G_0$ we have

$$|\varphi(x, z_1 + \lambda z_2) - \varphi(x, \lambda z_2)| \le M\alpha(z_1) = 0.$$

Hence $\varphi(x, z_1 + \lambda z_2) = \varphi(x, \lambda z_2)$ for $|\lambda| < \epsilon,\ x \in G_0$. From this it follows that

$$\varphi(x, z_1 + \lambda z_2) = \varphi(x, \lambda z_2) \text{ for all } \lambda \text{ and } x \in G.$$

Hence $\varphi(x, z_1 + z_2) = \varphi(x, z_2)$ for $x \in G$ and $z_1 \in\mathrm{Ker}\ \alpha$. Thus φ may be considered as a plurisubharmonic function on $G \times E^\alpha$. From what we have proved above, it follows that there exists a neighbourhood V of zero in E^α and a plurisubharmonic function g on $G \times E_V^\alpha$ such that

$$\varphi(x, z_2) = g(x, \Pi_V^\alpha(z_2)), \tag{2.1}$$

where $\Pi_V^\alpha : E^\alpha \to E_V^\alpha$ is the canonical projection. Put $U = \mathrm{Ker}\ \alpha \oplus V$. Then $E_U = E_V^\alpha$. Let

$$\Pi^\alpha : E \longrightarrow E^\alpha,$$
$$\Pi_U : E \longrightarrow E_U = E_V^\alpha$$

be the canonical projections. We have

$$\Pi_U = \Pi_V^\alpha \circ \Pi^\alpha \tag{2.2}$$

Combining (2.1) and (2.2), we get

$$\varphi(x,z)=\varphi(x,\Pi_U(z)). \tag{2.3}$$

Thus, φ is uniformly plurisubharmonic on $G\times E$.

b) Let E be a nuclear Fréchet space such that every plurisubharmonic function of the class C^1 on $G\times E$ is of uniform type. To prove the property $\overline{\overline{\Omega}}$ of E, it is sufficient by Vogt [9] to show that $L(E,H(\Delta))=LB(E,H(\Delta))$, where Δ is the unit ball in $\mathbb{C}$, $L(E,H(\Delta))$ is the space of continuous linear maps of E into $H(\Delta)$, and $LB(E,H(\Delta))$ the subspace of $L(E,H(\Delta))$ consisting of maps which are bounded on a neighbourhood of zero in E. Take an arbitrary element $T\in L(E,H(\Delta))$. Define a holomorphic function $\hat{T}:\Delta\times E\to\mathbb{C}$ by $\hat{T}(x,z)=T(z)[x]$. Then the function $(x,z)\to|\hat{T}(x,z)|^2$ is plurisubharmonic and belongs to the class C^1 on $\Delta\times E$. By the assumption, there exists a continuous seminorm p on E and a plurisubharmonic function

$$\Phi:\Delta\times E_p\mapsto[-\infty,\infty)$$

such that

$$|\hat{T}(x,z)|^2=\Phi(x,\Pi_p(z)) \quad \text{for all} \quad z\in E.$$

From the linearity of T, it follows that

$$\Phi(x,\lambda z)=|\lambda|^2\Phi(x,z) \quad \text{for all} \quad z\in E_p$$

Let

$$\Delta=\bigcup_{i=1}^{\infty}K_i,$$

where $\{K_j\}$ is a compact exhaustion sequence of Δ. By the upper-semicontinuity of Φ, there is a neighbourhood V_i of zero in E_p such that Φ is bounded on each $K_i\times V_i$. We shall show that Φ is bounded on $K\times V_1$ for each compact set K in Δ. Obviously, $K\subset K_i$ and $V_1\subset\lambda V_i$ for some i and λ.

We have

$$\sup_{\substack{x\in K\\ z\in V_1}} \Phi(x,z) \le \sup_{\substack{x\in K_i\\ z\in V_1}} \Phi(x,z) \le \sup_{\substack{x\in K_i\\ z\in V_i}} \Phi(x,\lambda z) \le$$
$$\le \lambda^2 \sup_{\substack{x\in K_i\\ z\in V_i}} \Phi(x,z) < \infty$$

Put $U_1 = \Pi_p^{-1}(V_1)$. Then U_1 is a neighbourhood of zero in E and we get

$$\sup_{z\in U_1} \sup_{x\in K} |T(z)[x]|^2 \le \sup_{\substack{x\in K\\ z\in U_1}} |\hat{T}(x,z)|^2 \le \sup_{\substack{x\in K\\ z'\in V_1}} \Phi(x,z') < \infty.$$

Thus $T \in LB(E < H(\Delta))$. Theorem 2.1 is now completely proved.

Under the assumption that E is a nuclear Fréchet space, it follows from Theorem 2.1 that if E does not have the property $\overline{\overline{\Omega}}$, there is a plurisubharmonic function on $G \times E$ which is not of uniform type (in the sense of our definition). However, this does not happen in the case of DFC-spaces. Recall that a DFC-space is any space of the form $E = F_c'$ where F is a Fréchet space, i.e. the dual of a Fréchet space endowed with the topology of compact convergence.

THEOREM 2.2. *Every plurisubharmonic function on a separable DFC-space is of uniform type.*

PROOF: Let E be a separable DFC-space and $\varphi \in PS(E)$. Set

$$U_j = \varphi^{-1}(-\infty, r_j) \quad \text{for} \quad r_j \in Q.$$

Then the family $\{U_j : r_j \in \mathrm{Q}\}$ forms an open covering of E. We can replace this family by a family $\{x_i + V_i\}_{i\in N}$ which satisfies the following conditions

a) V_i is a convex, balanced neighbourhood of zero.

b) For each i, there exists j such that $x_i + V_i \subset U_j$.

By [4], there exists a sequence $\{\lambda_j\}$ such that $U = \bigcap_{j=1}^{\infty} \lambda_j V_j$ is also a neighbourhood of zero in E. There is a plurisubharmonic function

$$\tilde{\varphi} : E/\mathrm{Ker}\ P_U \to [-\infty, \infty)$$

such that $\varphi = \tilde{\varphi} \circ \Pi_U$. On the other hand, by the assumption we can write $E = \bigcup_{n=1}^{\infty} K_n$, where K_n is compact in E with $nK_n \subset K_{n+1}$ for $n = 1, 2, \ldots$. For each n, there exists a neighbourhood W_n of zero such that

$$\sup\{\varphi(x) : x \in K_{n+1} + W_n\} < \infty.$$

Let $W = \bigcap_{n=1}^{\infty} \{K_n + \frac{1}{n} W_n\}$, where W_n are choosen such that $W_n \subset W_{n+1}$, $n = 1, 2, \ldots$. Since E is a DFC-space, W is also open in E. We have $nW \subset K_{n+1} + W_n$. Thus

$$\sup\{(x) : x \in nW\} \leq \sup\{\varphi(x) : x \in K_{n+1} + W_n\} < \infty$$

Put $D = W \cap U$. Then φ is bounded on nD for all $n \geq 2$. Hence $\tilde{\varphi} \circ \Pi$ is plurisubharmonic and bounded on every nD, $n \geq 2$, where Π is the restriction of the canonical projection $\Pi_{DU} : E_D \to E_U$ on the space $E/\text{Ker } P_D$. Define a function g on E_D by

$$g(z) = \limsup_{\substack{z' \to z \\ z' \in E/\text{Ker } P_D}} \tilde{\varphi} \circ \Pi(z')$$

Then g is plurisubharmonic on E_D [7] and $\varphi = g \circ \Pi_D$.

The proof of Theorem 2.2 is complete.

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REFERENCES

1. S. Deneen, R. Meise and D. Vogt, *Characterization of nuclear Fréchet space in which every bounded set is polar*, Bull. Soc. Math. France **112** (1984), 41–68.
2. C. Matyszczyk, *Approximation of analytic and continuous mapping by polynomials in Fréchet spaces*, Studia Math. **60** (1977), 223–238.
3. R. Meise and D. Vogt, *Holomorphic functions of uniform bounded type on nuclear Fréchet spaces*, Studia Math. **85** (1986), 147–166.
4. G. Mujica, *Domains of holomorphy on DFC spaces*, Lecture Notes in Math. **834** (1981), 500–532.
5. G. Mujica, *Holomorphic approximation in infinites dimensional Riemann domains*, Studia Math. **84** (1985), 107–134.

6. P. Noverraz, *approximation of holomorphic or plurisubharmonic functions in certain Banach spaces*, In : Proceedings on Infinite Dimensional Holomorphy, Lectute Notes in Math. **364** (1974), 175–185.
7. P. Noverraz, *Pseudocomplex completion of locally convex topological vector spaces*, Math. Ann. **208** (1974), 59–69.
8. Bui Dac Tac, *The Oka-Weil theorem in vector topological spaces*, (To appear in Ann. Pol. Math.).
9. D. Vogt, *Frechetträume zwischen denen jede stetige lineare Abbildung beschränkt ist*, J. Reine Angew. Math. **345** (1983), 182–200.

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