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ON THE EXISTENCE OF SOLUTIONS FOR FUNCTIONAL DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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1. Introduction

The aim of this paper is to prove the existence theorem for local and global solutions of infinite-dimentional functional differential inclusion(FDI).

The problem of the existence of solutions in multivalued differential equations and their properties has been the subject for a large amount of works of the last decade. It suffices to refer to the monographs [2] and [10] (which also contain extensive lists of references on this topic) for an overview on this area of research. It is worth noticing, however, that most literature was devoted to ordinary differential inclusions and quite a few and scattered results (see, e. g. [7], [9]) have been know for other types of differential inclusions like FDI or differential inclusions with retarded arguments.

In this paper, using the approach developed by C.Castaing and M. Valadier [4] we are able to establish sufficient conditions for the existence of local and global solutions of FDI in Banach spaces under quite general assumptions.

For the sake of convenience, we list some notations used in this paper. Throughout the paper, E denotes a separable Banach space with the norm and the strong dual E', E_σ and E'_s are the spaces E, and E' endowed with the weak topologies $\sigma(E, E')$ and $\sigma(E', E)$ respectively.

For $T > 0$ and $h \geq 0$, $\mathcal{C}_E[-h, T]$ and $\mathcal{C}_{E_\sigma}[-h, T]$ stand for the spaces of continuous functions from $[-h, T]$ to E and to E_σ, respectively. It is obvious that $\mathcal{C}_E[-h, T] \subset \mathcal{C}_{E_\sigma}[-h, T]$. We shall endow $\mathcal{C}_E[-h, T]$ and $\mathcal{C}_{E_\sigma}[-h, T]$

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with the topology of uniform convergence on $[-h,T]$. Recall that the neighbourhood base of $\mathcal{C}_{E_\sigma}[-h,T]$ consists of sets of the form $\{f \in \mathcal{C}_{E_\sigma}[-h,T] : f([-h,T]) \subset V\}$, where$V$ is a neighbourhood of the origin in E_σ. We shall denote by $\mathcal{L}^1_E[0,T](L^1_E[0,T])$ the space of all integrable (resp., equivalent classes of integrable) functions from $[0,T]$ into $\mathcal{L}^1[0,T] = \mathcal{L}^1_R[0,T]$; and by $L^\infty_{E'_s}[0,T]$ the space of all classes of essentially bounded measurable functions from $[0,T]$ into E'_s; $L^\infty_{[0,T]} = L^\infty_R[0,T]$. Throughout, the integral is understood in the sense of Bochner.

Finally, let $\Gamma : [0,T] \to 2^E$ be a measurable multifunction such that, for each $t \in [0,T]$, $\Gamma(t)$ is a nonempty convex $\sigma(E,E')$ - compact subset of E . We shall denote by $\mathcal{S}_\Gamma$ (resp., S_Γ) the set of all measurable selections (resp., the set of all equivalent classes of measurable selections) of Γ. We shall suppose that Γ is integrable, that is there exists a positive integrable function $m(\cdot)$ such that $\| x \| \leq m(t)$ for every $t \in [0,T]$ and every $x \in \Gamma(t)$. Then, clearly, $\mathcal{S}_\Gamma \subset \mathcal{L}^1_E[0,T]$. By definition, for $t,t' \in [0,T]$,

$$\int_t^{t'} \Gamma(s)\, ds = \{\int_t^{t'} f(s)\, ds : f \in \mathcal{S}_\Gamma\}.$$

It is important to note that the concepts of measurability and weak or scalar measurability for functions from $[0,T]$ into E coincide because E is a separable Banach space (see, e. g.[4],[11]).

2. Existence of local solutions for FDI

Let U be a given neighbourhood of the origin in E_σ. Let us define

$$D = \{\varphi \in \mathcal{C}_E[-h,0] : \varphi([-h,0]) \subset U\}$$

Note that D is an open set in $\mathcal{C}_E[-h,0]$ and, therefore, D is a Suslin space.Let $G : [0,T] \times D \to 2^E$ be a multifunction with nonemty convex

$\sigma(E,E')$-compact values in E. For a given function $\varphi^0 \in D$, we consider the functional differential inclusion (FDI) of the form

$$\dot{x}(t) \in G(t, x_t), \quad t \in [0,T], \tag{2.1}$$

$$x(\theta) = \varphi^0(\theta), \quad \theta \in [-h, 0], \tag{2.2}$$

where,by definition, $x_t(\theta) = x(t+\theta)(\forall t \in [0,T], \forall \theta \in [-h,0])$. We shall say that a function $x \in \mathcal{C}_E[-h,T]$ is a local solution of FDI (2.1) satisfying the initial condition (2.2) if there exists $T_0 \in (0,T]$ such that $x(\cdot)$ is absolutely continuous on $[0,T]$ and satisfies the inclusion(2.1) a.e. on$[0,T_o]$ and (2.2). If $T_0 = T$ then x is said to be a global solution.

THEOREM2.1. *Let Γ and G be multifunctions satisfying the above hypotheses. Moreover,we assume that*

(i) for every $t \in [0,T]$ and every $\varphi \in D$, $G(t,\varphi) \subset \Gamma(t)$;

(ii) for every $\varphi \in D$, $G(.,\varphi)$ is measurable on $[0,T]$;

(iii) for every $t \in [0,T]$, $G(t,.)$ is an upper-semicontinuous(u.s.c) function from D to E_σ.

Then for every $\varphi^0 \in D$, the set of local solutions of FDI (2.1) is nonempty.

PROOF: . Since $\varphi^0 \in D$, $\varphi^0(0) \in U$ and, therefore, there is a neighbourhood V of the origin for the weak topology $\sigma(E,E')$ such that $\varphi^(0)+V \subset U$. Since the multifunction $t \to \int_0^t \Gamma(s)\, ds$ is u.s.c. from $[0,T]$ into E_σ (see Th.II-20 in [4]), there exists $T_0 > 0$ such that $\int_0^t \Gamma(s)\, ds \subset V \quad (\forall t \in [0,T_0])$. So we have

$$\varphi^0(0) + \int_0^t \Gamma(s) \subset U, \forall t \in [0,T_0]. \tag{2.3}$$

Let us define a set of functions

$$X = \{x(\cdot) \in \mathcal{C}_E[-h, T_0] : x(\theta) = \varphi^0(\theta), \forall\theta \in [-h, 0],$$
$$x(t) = \varphi^0(0) + \int_0^t \sigma(s)ds, \forall t \in [0, T_0], \sigma \in \mathcal{S}_\Gamma\}$$

It is clear that X is a nonempty and convex subset of $\mathcal{C}_E[-h, T_0]$. We will show that X is compact when regarded as a subset of $\mathcal{C}_{E_\sigma}[-h, T_0]$. Observe first that for each $t \in [0, T_0]$, $\int_0^t \Gamma(s)\,ds$ is a compact subset of E_σ. Therefore, by ascoli's theorem, it suffices to prove that X is uniformly equicontinuous, i.e., for any 0-neighbourhood V in E_σ, there exists $\delta > 0$ such that for every $x \in X$,

$$x(t) - x(t') \in V$$

whenever $|t - t'| < \delta$ and $t, t' \in [-h, T_0]$. Without loss of generality, we may take $V = V_{e',\varepsilon} = \{x \in E : | < e', x > | < \varepsilon\}$ with some nonzero $e' \in E'$ and $\varepsilon > 0$. Since φ^0 is uniformly continuous on $[-h, 0]$ and $\int_0^t m(s)ds$ is absolutely continuous on $[0, T_0]$, there is $\delta = \delta(\varepsilon) > 0$ such that for $t, t' \in [-h, T_0]$ with $|t - t'| < \delta$, we have

$$\int_t^{t'} m(s)ds < \frac{\varepsilon}{2}||e'||, \text{ if } t, t' \in [0, T_0];$$

and

$$||\varphi^0(t) - \varphi^0(t')|| < \frac{\varepsilon}{2}||e'||, \text{ if } t, t' \in [-h, 0].$$

It follows that for any $x \in X$ and for any $t, t' \in [-h, T_0]$ with $|t - t'| < \delta$, the following condition holds $| < e', x(t) - x(t') > | < \varepsilon$, or, equivalently, $x(t) - x(t') \in V$. Thus, X is an uniformly equicontinuous subset of $\mathcal{C}_{E_\sigma}[-h, T_0]$, the closeness of X can be proved analogously as in [4, Th. VI.1] and the compactness of X follows from Ascoli's theorem.

Now consider a multifunction $\Phi : X \to 2^X$ defined as

$$\Phi(x) = \{y \in X : \dot{y}(t) \in G(t, x_t)\text{a.e. on}[0, T]\}$$

We will show that Φ admits a fixed point in X. Notice first that by virtue of (2.3),for each $x \in X$ and for each $t \in [0,T], x_t \in D$. Morever, it is obvious that the function $t \to x_t$ is continuous from $[0,T_0]$ into D. Therefore, by [4, Th.VI-6], there exists a measurable function $\sigma : [0,T_0] \to E$ such that $\sigma(t) \in G(t, x_t)$ a.e. on $[0,T_0]$. In view of (i), $\sigma \in \mathcal{S}_\Gamma$. We set

$$y(t) = \begin{cases} \varphi^0(t), \text{ for } t \in [-h,0] \\ \varphi^0(0) + \int_0^t \sigma(s)\, ds, \text{ for } t \in [0,T_0] \end{cases}$$

It is clear that $y \in \Phi(x)$. Thus, $\Phi(x)$ is a nonempty and convex subset for each $X \in X$. In order to appply the Kakutani-Ky Fan fixed point theorem, it remains to show that Φ is u.s.c., or, equivalently ([3]), that the graph of Φ is closed in $X \times X$ (with respect to the induced topology of $\mathcal{C}_{E_\sigma}[-h,T_0] \times \mathcal{C}_{E_\sigma}[-h,T_0]$). To this end, noticing that $X \times X$ is metrizable, we suppose that $\{(x^k, y^k)\}_{k=1}^{\infty}$ is a sequence in the graph of Φ converging to $(x,y) \in X \times X$. Then, by definition, $\dot{y}^k(t) \in G(t, x_t^k)$ a.e. on$[0,T]$ and, for any nonzero $e' \in E'$ and for any $\varepsilon > 0$, there exists N such that

$$|< e', x^k(t) - x(t) >| < \varepsilon \quad (\forall k \geq N, \forall t \in [-h,T_0]).$$

Therefore,

$$|< e', x_t^k(\theta) - x_t(\theta) >| < \varepsilon \quad (\forall k \geq N, \forall \theta \in [-h,0]).$$

In other words, x_t^k converges to x_t in the topology of $\mathcal{C}_{E_\sigma}[-h,0]$. On the other hand, let

$$y^k(t) = \begin{cases} \varphi^0(t) & \text{if } t \in [-h,0], \\ \varphi^0(0) + \int_0^t \sigma_k(s)\, ds & \text{if } t \in [0,T_0], \end{cases}$$

and

$$y(t) = \begin{cases} \varphi^0(t) & \text{if } t \in [-h,0], \\ \varphi^0(0) + \int_0^t \sigma(s)\, ds & \text{if } t \in [0,T_0], \end{cases}$$

where σ_k and σ belong to $\mathcal{S}_\Gamma$. Since y^k converges to y in $C_{E_\sigma}[-h, T_0]$, it follows that for any $e' \in E'$,

$$(2.4) \qquad \int_0^t < e', \sigma_k(s) > \ ds \to \int_0^t < e', \sigma(s) > \ ds \ \ (\forall t \in [0, T_0]).$$

Since the set S_Γ (the quotient of $\mathcal{S}_\Gamma$ for the equivalence " equality a.e. ") is compact for the weak topology $\sigma(L^1_E[0, T_0], L^\infty_{E'_s}[0, T_0])$,there exists a subsequence $\{\sigma_{ki}\}_{i=1}^\infty$ which converges to $\bar{\sigma} \in \mathcal{S}_\Gamma$. This implies that for each $e' \in E'$, the sequence $\{< e', \sigma_{ki}(\cdot) >\}_{(i=1)}^\infty$ converges to $< e', \sigma(\cdot) >$ for the weak topology $\sigma(L^1_{[0,T]}, L^\infty_{[0,T_0]})$. Hence, we can apply Theorem VI.4 in [4] to conclude that $\sigma(t) \in G(t, x_t)$ a.e. on $[0, T_0]$. Moreover, from the above it follows that

$$\int_0^t < e^z[z'], \sigma_(ki)(s) > ds \to \int_0^t < e', \ \sigma(s) > ds,$$

for each $t \in [0, T_0]$ and for each $e' \in E'$. This gives, in view of (2.4),

$$< e', \int_0^t \sigma(s) > = < e', \int_0^t \ \sigma ds >, \ \ (\forall t \in [0, T_0]).$$

Since E is a separable Banach space, it follows that

$$\int_0^t \sigma ds = \int_0^t \bar{\sigma}(s) ds, \ \ (\forall t \in [0, T_0]).$$

Consequently, $y \in \Phi(x)$. Thus the graph of Φ is a closed subset of $X \times X$. According to Kakutani-Ky Fan's theorem, Φ admits a fixed point $\bar{x} \in X$, i.e.$\bar{x} \in \Phi(\bar{x})$. Clearly, $\bar{x}$ is a solution of (2.1) satisfying the innitial condition (2.2). Theorem 2.1 is completely proved.

3. Existence of global solution for FDI

Let $G : [0, T_o] \times C_{E_\sigma}[-h, 0] \to 2^E$ be a multifunction with nonempty convex compact values in E_σ. For given $\varphi^0 \in C_E[0, T_0]$, we consider the following FDI

$$\dot{x}(t) \in G(t, x_t), \quad t \in [0, T], \tag{3.1}$$

with the initial condition

$$x(\theta) = \varphi^0(\theta), \quad \theta \in [-h, 0]. \tag{3.2}$$

THEOREM 3.1. . *Suppose that G satisfies the following assumptions:*

(i) for every $\varphi \in \mathcal{C}_{E_\sigma}[-h,0], G(.,\varphi)$ is measurable on $[0,T]$;

(ii) for every $t \in [0,T], G(.,\varphi)$ is a u.s.c. multifunction from $\mathcal{C}_{E_\sigma}[-h,0]$ into E_σ;

(iii) there exits a balanced convex $\sigma(E,E')$ - compact set $K \subset E$ and exists a positive function $\alpha(\cdot) \in \mathcal{L}^1_{[0,T]}$ such that for every $t \in [0,T]$ and for every $\varphi \in \mathcal{C}_E[-h,0]$,

$$G(t,\varphi) \subset \alpha(t)(1+ \| \varphi \|)K.$$

Then, for every $\varphi^0 \in \mathcal{C}_E[-h,0]$, the set of global solutions of (3.1) is nonempty.

PROOF: Set

$$c = sup\{\| x \| : x \in K\},$$

and

$$a = \| \varphi^0 \| := sup\{\| \varphi^0(\theta) \| : \theta \in [-h,0]\}.$$

Suppose that $x(\cdot)$ is a global solution of (3.1) satisfying (3.2), then we have $x(\theta) = \varphi^0(\theta), \forall t \in [-h,0]$ and $x(t) = \varphi^0(0) + \int_0^t \dot{x}(s)ds(\forall t \in [0,T]$ with $\dot{x}(t) \in G(t,x_t)$ a.e. on $[0,T]$. Therefore, for every $t \in [-h,T]$,

$$\| x(t) \| \leq a + \int_0^t \| \dot{x}(s) \| \leq a + c \int_0^t \alpha(s)(1+ \| x_s \|)ds.$$

This implies

$$\| x(t+\theta) \| \leq a + c \int_0^t \alpha(s)(1+ \| x_s \|)ds$$

for all $\theta \in [-h, 0]$, and therefore,

$$\| x_t \| \leq a + c \int_0^t \alpha(s)(1+ \| x_s \|)ds, \quad \forall t \in [0, T].$$

Using Gronwall's lemma, we obtain that every solution $x(\cdot)$ of (3.1)-(3.2) must satisfy the following inequality

$$\| x_t \| \leq (a+i) exp(c \int_0^t \alpha(s)ds) - 1, \quad \forall t \in [0, T].$$

Put $z(t) = (a+1)exp(c \int_0^t \alpha(s)ds) - 1, t \in [0, T]$. It is easy to verify that $z(t)$ is the unique solution of the equation

$$z(t) = a + \int_0^t \alpha(s)(1 + z(s))ds$$

(the uniqueness follows from Gronwall's lemma). Now let us define the multifunction

$$\Gamma(t) + \alpha(t)(1 + z(t))K.$$

Then, by [4, Corollary V.4] and [1], the quotient S_Γ of the set $\mathcal{S}_\Gamma$ of all measurable selections of Γ is a nonempty convex set of $L^1_E[0, T]$ which is compact for the weak topology $\sigma(L^1_E[0, T], L^\infty_{E'_s}[0, T])$. Therefore, S_Γ is metrizable.

For every $f \in \mathcal{S}_\Gamma$, we set

(3.3)
$$x^f(t) = \begin{cases} \varphi^0(t) \text{ for } t \in [-h, 0] \\ \varphi^0(0) + \int_0^t f(s)ds \text{ for } t \in [0, T] \end{cases}$$

Obviously, $x^f(\cdot) \in C_E[-h,T]$. Let us introduce the multifunction ϕ by setting, for each $f \in S_\Gamma$,

$$\Phi(f) = \{g : [0,T] \to E : g \text{ measurable and } g(t) \in G(t, x_t^f)\}.$$

Since the function $t \to x_t^f$ is continuous from $[0,T]$ into $C_{E_\sigma}[-h,0]$, it follows from [4, Corollary VI-5] that $\Phi(f)$ is nonempty for every $f \in S_\Gamma$. Further, by the definition of Γ, for each $f \in S_\Gamma$ we have

$$\| f(t) \| \leq \alpha(t)(1+z(t))c, \ \forall \in [0,T],$$

and hence,

$$\begin{aligned} \| x^f(t) \| &\leq a + \int_0^t \| f(s) \| ds \\ &\leq a + c\int_0^t \alpha(s)(1+z(s))ds = z(t), \ \forall t \in [0,T]. \end{aligned}$$

This implies that , for every $g \in \Phi(f)$ and a.e. $t \in [0,T]$,

$$g(t) \in G(t, x_t^f) \subset \alpha(t)(1+ \| x_t^f \|)K = \Gamma(t).$$

Consequently, $\Phi(f) \subset S_\Gamma$, $\forall f \in S_\Gamma$. Thus, Φ is a multifunction with nonempty convex values from a compact metrizable set S_Γ into itself. To apply the fixed point Kakutani- Ky Fan theorem, it suffices to show that the graph of Φ is a closed subset of $S_\Gamma \times S_\Gamma$. Suppose that a sequence $\{(f_k, g_k)\}_{k=1}^\infty$ belonging to the graph of Φ converges in $S_\Gamma \times S_\Gamma$ to (f,g). Then, by definition, $g_k(t) \in G(t, x_t^{f^k})$ for a. e. $t \in [0,T]$. It is clear that for every $e' \in E'$, the sequence $\{< e', f_k(\cdot) >\}_{k=1}^\infty$ (resp., to $\{< e', g_k(\cdot) > \}_{k=1}^\infty$) converges to $< e', f(\cdot) >$ (resp., to $< e', g(\cdot) >$) for the weak topology $\sigma(L^1[0,T], L^\infty[0,T])$. In particular, for every $\varepsilon \geq 0$ and for every $t \in [0,T]$, there exists $N = N(t,\varepsilon)$ such that

$$\left| \int_0^t < e', f_k(s) - f(s) > ds \right| \leq \frac{\varepsilon}{2},$$

whenever $k \geq N$.

Taking $\delta = \delta(t) > 0$ sufficiently small, we can write, for all $t' \in [t-\sigma, t+\delta] \cap [0,T]$ and for all $k \geq N$, the following estimate

$$|\int_0^{t'} < e', f_k(s) - f(s) > ds| \leq |\int_0^{t} < e', f_k(s) - f(s) > ds|$$

$$+|\int_t^{t'} < e', f_k(s) - f(s) > ds| \leq \frac{\varepsilon}{2} + 2 \| e' \| c \int_t^{t'} \alpha(s)(1 + z(s)ds$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Let $\{t \in [0,T] :| t - t_i | \leq \delta_i\}, i = 1, 2, ..., m$, be a finite covering for$[0,T]$ and let us define $N_\varepsilon = max\{N(t_i, \varepsilon), i = 1, 2, ..., m\}$. Then, for all $t \in [0,T]$.

$$| \int_0^{t} < e', f_k(s) - f(s) > ds | < \varepsilon$$

wherever $k \geq N_\varepsilon$. Consequently, we obtain that

$$| < e', x_t^{f_k}(\theta) - x_t^{f}(\theta) > | < \varepsilon, \quad \forall t \in [0,T], \quad \forall \theta \in [-h, 0],$$

whenever $k \geq N_\varepsilon$. This means that $x_t^{f_k}$ converges to x_t^f in $\mathcal{C}_{E_\sigma}[-h, 0]$ for the topology of uniform convergence. Therefore, by [4, Th.VI-4], it follows that $g(t) \in G(t, x_t^f)$ a.e. on $[0,T]$, and, hence, $g \in \Phi(f)$ or, equivalently, (f,g) belongs to the graph of Φ. Thus, by Kakutani-Ky Fan's theorem, Φ admits a fixed point $f_0 \in S_\Gamma$. The function $x^{f_0}(\cdot)$, defined by (3.3), is then a global solution of FDI (3.1) satisfying the initial condition (3.2). The proof is complete.

4. A particular case : Retarded differential inclusions

The functional differential inclusion (2.1) is a very general type of inclusions and includes, as particular cases, ordinary differential inclusions

$$\dot{x}(t) \in F(t, x(t)),$$

retarded differential inclusions

$$\dot{x}(t) \in F(t, x(t - r_1(t)),\ x(t - r_2(t)), \cdots, x(t - r_p(t))),$$

with $0 \le r_i(t) \le h,\ i = 1, 2, \cdots, p$, as well as the integral-differential inclusion

$$\dot{x}(t) \in \int_{-h}^{0} F(t, \theta, x(t + \theta))d\theta.$$

To illustrate the above result let us consider the case of retarded differential inclusions. Let U be an open subset of E_σ, φ^0 a given strongly continuous function from $[0, T]$ into U and $r_i(\cdot)\ (i = 1, \cdots, p)$ given continuous functions from $[0, T]$ into $[0, h]$.

Suppose that $\Gamma : [0, T] \to 2^E$ is an integrable multifunction with nonempty convex $\sigma(E, E')$-compact values in E and $F : [0, T] \times U^p \to 2^E$ is a multifunction with nonempty convex $\sigma(E, E')$-compact values in E satisfying the following conditions

i) for every $t \in [0, T]$ and for every $(x^1, x^2, \cdots, x^p) \in U^p$,

$$F(t, x^1, x^2, \cdots, x^p) \in \Gamma(t);$$

ii) for every $(x^1, x^2, \cdots, x^p) \in U^p$, $F(., x^1, x^2, \cdots, x^p)$ is measurable on $[0, T]$;

iii) for every $t \in [0, T]$, $F(t, \cdot)$ is u.s.c. from U^p into E_σ.

Then the retarded differential inclusion

(4.1) $$\dot{x}(t) \in F(t, x(t - r_1(t)), x(t - r_2(t)), \cdots, x(t - r_p(t))),\ t \in [0, T],$$

with the initial condition

(4.2) $$x(\theta) = \varphi^0(\theta),\ \ \theta \in [-h, 0],$$

has a solution defined on $[-h, T_0]$ with $0 < T_0 \leq T$. To prove this, it suffices to set

$$D = \{\varphi \in \mathcal{C}_E[-h, 0] : \varphi([-h, 0]) \subset U\}$$

and define a multifunction $G : [0, T] \times D \to 2^E$ by

$$G(t, \varphi) = F(t, \varphi(-r_1(t)), \varphi(-r_2(t)), \cdots, \varphi(-r_p(t))).$$

The result is now immediate from the above Theorem 2.1.

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