ACTA MATHEMATICA VIETNAMICA

Vol. 14 No 2 (1989)

ON THE EXISTENCE OF AN OPTIMAL CONTROL FOR A STOCHASTIC OPTIMIZATION PROBLEM WITH CONSTRAINTS

TRUONG CHI TIN

INTRODUCTION

This paper deals with discrete — time stochastic optimization problem with constraints. The main result of this paper is Theorem 3. 1 which gives sufficient conditions for the existence of an optimal control for this problem. The proof of the main result is based on the dynamic programming approach. An application of this result to the problem of the existence of optimal control for the hydroelectric station « Hoa Binh » will be shown,

Let $(\Omega, \mathcal{F}, P)$ be a basic probability space. Let us consider an object whose dynamics is given by the following system of equations:

$$\begin{cases} x_{n+1} = f_n(x_n, u_n, q_{n+2}) \\ x_0 = x_0,\ n = 0,1 ..., N-1 \end{cases} \tag{0.1}$$

with constraints.

$$u_n \in U_n (\subset U) \tag{0.2}$$

$$P\{x_{n+1} \in X_{n+1}\} = 1,\ n = 0,1 ..., N-1 \tag{0.3}$$

where $\{q_1, q_2, ..., q_N\}$ is the perturbation process with values in $\{Q_1, Q_2, ..., Q_N\}$ $(\subset R^d)$; the state $\{x_0, x_1, ..., x_N\}$ and the control policy $\{u_0, u_1, ..., u_{N-1}\}$ are stochastic processes in R^n and R^m respectively; $\{X_k\}$, $\{U_k, U\}$, $\{Q_k\}$ are closed subsets of R^n, R^m and R^d respectively; the functions $f_k: R^n \times U_k \times R^d \to$ $\to R^n$ $(k = 0,1,..., N-1)$ are Borel measurable.

The control $u = \{u_k, k = 0,1,..., N-1\}$ is called admissible if $u_0 = g_0(x_0)$, $u_i = g_i(x_i, q_i)$ $(i = 1, 2,..., N-1)$, where $g_0: X_0 \to U_0$, $g_i: X_i \times Q_i \to U_i$ are Borelian functions and the corresponding solution of (0. 1) satisfies the constraints (0.3). The set of these control is denoted by $\mathcal{U}$.

If $\mathcal{U} \neq \phi$, then for every $u \in \mathcal{U}$ we consider the cost functional of the form:

$$J(x_0, u) = \mathrm{E}\,\{h_0(x_0, u_0) + \sum_{n=1}^{N-1} h_n(x_n, u_n, q_n) + h_N(x_N, q_N)\}, \tag{0.4}$$

where $\{h_n\}$ are Borelian functionals bounded from below. Ey is the mathematical expectation of the random variable y.

An admissible control $\widehat{u}$ is called optimal if

$$J(x_0, \widehat{u}) = \inf_{u \in \mathcal{U}} J(x_0, u).$$

In [1], [3], [4], V. I. Arkin and L.I. Krechelov have given a necessary condition for an optimal control for this problem by means of the stochastic maximum principle. But this necessary optimality condition and the conditions ([3]), under which the class of admissible policies is non—empty, are practically difficult to verify. Recently, in [2] F.J. Beutler and K.W. Ross have studied optimal policies for controlled Markov chain only for the case where the state space contains a finite number of elements.

In this paper this restriction will be eliminated. In Section I, we shall give some conditions for $\mathcal{U}$ to be non-empty. Section 2 is devoted to the formulation of a necessary and sufficient condition for the optimality of a control by means of the equation of dynamic programming. In Section 3, we shall prove the existence of an optimal control and give an application to the problem of optimal control for the hydroelectric station «Hoa Binh» in the North of Vietnam.

1. EXISTENCE OF ADMISSIBLE CONTROLS

THEOREM 1.1

i) *Assume that for every* $n = 0, 1, \ldots, N-1$, U_n *is compact, the function* $f_n(x, v, q)$ *is continuous with respect to* $(x, v) \in X_n \times U_n$ *and* $\forall\, x \in X_n\ \exists\, v \in U_n$: $f_n(x, v, q_{n+1}) \in X_{n+1} (P - a.s.)$. *Then* $\mathcal{U} \neq \phi$.

ii) *Conversely, if there exists an admissible control* $u \in \mathcal{U}\ (\neq \phi)$ *and* $\{x_n\}$ *is the corresponding solution of (0. 1),* Q_n $(n = 1, 2, \ldots, N)$ *are discrete spaces, then*

$\forall n = 0, 1, \ldots, N-1\ \exists X'_n \subset X_n : P\ \{x_n \in X'_n\} = 1$ *and* $\forall\, x \in X'_n\ \exists\, v \in U_n$:

$f_n(x, v, q) \in X'_{n+1}$ $(\forall\, q \in Q_{n+1})$.

Proof. i) For every $n = 0, 1, \ldots, N-1$, let us consider the set-valued map F_n from X_n to U_n defined by

$$F_n(x) = \{v \in U_n : f_n(x, v, q_{n+1}) \in X_{n+1}\ (P - a.s.)\}.$$

Since X_{n+1} is closed and f_n is continuous, it follows that $F_n(x)$ is closed and non-empty. Indeed, let $\{v_k\}$ $(\subset E_n(x)) \to v$ as $k \to \infty$. Set: $\Omega_k :=$

$= \{\omega \in \Omega : f_n(x, v_k, q_{n+1}(\omega)) \in X_{n+1}\}$, $\Omega_0 : \bigcap_{k=1}^{\infty} \Omega_k$. Then $P(\Omega_k) = P(\Omega_0) = 1$

$\forall\, k \in \mathbb{N}$. Furthermore, for every $\omega \in \Omega_0$

$$f_n(x, v, q_{n+1}(\omega)) = \lim_{k \to \infty} f_n(x, v_k, q_{n+1}(\omega)) \in X_{n+1}; \text{ i. e. } v \in F_n(x).$$

We now prove that the set-valued map F_n is upper semicontinuous on X_n. For each closed subset E of U_n, put $Y_n := \{x \in X_n : F_n(x) \cap E \neq \phi\}$. Consider any sequence $\{y_k\}$ $(\subset Y_n) \to \widehat{y}$, $k \to \infty$. Clearly, $\widehat{y} \in X_n$. Let $v_k \in F_n(y_k) \cap E$ $(\forall k \in \mathbb{N})$. Since U_n is compact, without loss of generality we may assume that $v_k \to \widehat{v} \in U_n \cap E$. It follows easily that $\widehat{v} \in F_n(\widehat{y}) \cap E$, i. e. $\widehat{y} \in Y_n$. Hence Y_n is closed.

From the selection theorem [6] it follows that there exists a Borelian function $g_n : X_n \to U_n$ such that $g_n(x) \in F_n(x)$ $\forall x \in X_n$ $(n = 0,1,..., N-1)$. Hence there exists the admissible control $u = \{u_n = g_n(x_n), n = 0,1,..., N-1\} \in \mathcal{U}$.

ii) It should be noted that if the spaces $\{Q_n\}$ are discrete, then for each $u \in \mathcal{U}$ so are the state spaces $\{X'_n\}$ of $\{x_n\}$. The second part of theorem is then easily proved.

COROLLARY 1.2. *Assume that for each $n = 0,1,..., N-1$ U_n is compact, the function $f_n(x, v, q)$ is continuous with respect to $(x, v) \in X_n \times U_n$ and $\forall x \in X_n$ $\exists v \in U_n : f_n(x, v, q) \in X_{n+1}$ $(\forall q \in Q_{n+1})$. Then $\mathcal{U} \neq \phi$.*

Remark.

The above results remain valid if we add the following constraints to the problem (0,1) — (0.4):

$$P\{r_n(x_n, u_n, q_{n+1}) \in Y_{n+1}\} = 1, \; n = 0,1,..., N-1$$

where Y_n is a closed set and $r_n(x, v, q)$ is continuous with respect to $(x, v) \in X_n \times U_n$ for all n,...

2. EQUATION OF DYNAMIC PROGRAMMING

In the rest of this paper we assume that $\{q_1, q_2, ..., q_N\}$ is a Markov chain with transition probabilities $\{P_n(dy, q), n = 2, 3,..., N\}$ and an initial probability $P_1(dy)$.

It should be noted, that in this case, $\{x_n, n = 0,1,..., N\}$ is not necessary a Markov chain but for $u_n = g_n(x_n, q_n)$ the process $\{(x_n, q_n), 0 \leqslant n \leqslant N\}$ is Markovian.

For each $k = 1,2,\dots, N-1$, $(x, q) \in X_k \times Q_k$, we denote by $\mathcal{U}^{(k)}_{x\,q}$ the set of all control policies $u^{(k)} = \{g_j,\ j=k,\ k+1,\dots,\ N-1\}$, where $g_k(x, q) \in U_k$ and for each $i = k+1, k+2, \dots, N-1$ $g_i : X_i \times Q_i \longrightarrow U_i$ is a Borelian function such that the solution of the corresponding system:

$$\begin{cases} x_{j+1} = f_j(x_j, g_j(x_j, q_j), q_{j+1}) \\ x_k = x,\ q_k = q,\ j = k,\ k+1,\ \dots,\ N-1 \end{cases}$$

satisfies the constraint: $P\{x_{j+1} \in X_{j+1},\ j = k,\ k+1,\dots, N-1\} = 1$.

$V_k(x) := \{v \in U_k : P\{f_k(x, v, q_{k+1}) \in X_{k+1}\} = 1\},\ \forall\, x \in X_k$ $(k = 0, 1,\dots, N-1)$.

If the conditions of Theorem 1.1 are satisfied then these sets are non-empty.

$$\overline{W}_N(x, q) := h_N(x, q)\ \forall\,(x, q) \in X_N \times Q_N$$

$$\overline{W}_k(x,q) := \inf_{u^{(k)} \in \mathcal{U}^{(k)}_{x,\,q}} E\left\{\sum_{n=k}^{N-1} h_n(x_n, g_n(x_n, q_n), q_n) + h_N(X_N, q_N)/q_k = q\right\}$$

$$\forall (x, q) \in X_k \times Q_k\ (k = 1, 2,\dots, N-1)$$

$$\overline{W}_0(x_0) := \inf_{u \in \mathcal{U}} J(x_0, u).$$

Then we have the following theorem.

THEOREM 2.1. *Assume that the conditions of Theorem 1.1 or those of Corollary 1.2 are satisfied and there exist functionals* $\{W_k(x,q), (x, q) \in X_k \times Q_k\}$ *satisfying the relations:*

$$W_N(x, q) = h_N(x, q)\ \forall\,(x, q) \in X_N \times Q_N,$$

$$W_k(x,q) = \inf_{v \in V_k(x)} \{h_k(x, v, q) + \int_{Q_{k+1}} W_{k+1}(f_k(x, v, y), y) P_{k+1}(dy, q)\}$$

$$\forall (x, q) \in X_k \times Q_k\ (k = 1, 2,\dots, N-1)$$

$$W_0(x_0) = \inf_{v \in V_0(x_0)} \{h_0(x_0, v) + \int_{Q_1} W_1(f_0(x_0, v, y), y)\, P_1(dy)\}.$$

Then i) $W_k(x, q) \leqslant \overline{W}_k(x, q)\ \forall (x, q) \in X_k \times Q_k\ (k = 1, 2,\dots, N)$

$$W_0(x_0) \leqslant \overline{W}_0(x_0).$$

ii) If the control $\widehat{u} = \{\widehat{u_0} = \widehat{g_0}(\widehat{x_0}), = \widehat{u_k} = \widehat{g_k}(\widehat{x_k}, q_k),$ $k = 1, 2,\dots, N-1\}$ $(\in \mathcal{U})$ *satisfies the relations:*

$$W_k(x, q) = h_k(x, \widehat{g_k}(x, q), q) + \int_{Q_{k+1}} W_{k+1}(f_k(x, \widehat{g}k(x, q), y), y) P_{k+1}(dy, q)$$

$$\forall (x, q) \in X_k \times Q_k\ (k, = 1, 2,\dots, N-1)$$

$$W_0(x_0) = h_0(x_0, \widehat{g_0}(x_0)) + \int_{Q_1} W_1(f_0(x^0, \widehat{g_0}(x_0), y), y) P_1(dy)$$

where $\{\widehat{x}_k, k = 0, 1, \ldots, N\}$ *is the corresponding solution of* (0. 1), *then* $\widehat{u}$ *is an optimal control.*

iii) Conversely, if $\widehat{u}$ *is an optimal control in the following sense:*

$$W_0(x_0) = J(x_0, \widehat{u})$$

$$W_k(x, q) = E\left\{\sum_{u=k}^{N-1} h_n(\widehat{x}_n, \widehat{g}_n, \widehat{x}_n, q_n), q_n) + h_N(\widehat{x}_N, q_N) / q_k = q\right\}$$

$\forall (x, q) \in X_k \times Q_k$ $(k = 1, 2, \ldots, N - 1)$,

then $\widehat{u}$ *satisfies the relation in part (ii)*

Proof. It is clear that $\forall\, u^{(k)} \in \mathcal{U}^{(k)}_{x,q}$

$$E\left\{\sum_{n=k}^{N-1} h_n(x_n, g_n(x_n, q_n), q_n) + h_N(x_N, q_N)/q_k = q\right\} \geqslant$$

$$\geqslant h_k(x, g_k(x, q), q) + E\{\overline{W}_{k+1}(f_k(x, g_k(x, q), q_{k+1}), q_{k+1})/q_k = q\}.$$

From the above inequality and the equality $\overline{W}_N(x, q) = W_N(x, q)$ the asser-
…ion i) follows by induction on $k = 0, 1, \ldots N-1$. Also, by induction it is easily
…en, that for each $k = 1, 2, \ldots, N$

$$\ldots_0(x_0) = E\{h_0(\widehat{x}_0, \widehat{g}_0(\widehat{x}_0)) + \sum_{n=1}^{k-1} h_n(\widehat{x}_n, \widehat{g}_n(\widehat{x}_n, q_n), q_n) + W_k(\widehat{x}_k, q_k)\}.$$

…e remaining part of the proof is obvious.

3. EXISTENCE OF AN OPTIMAL CONTROL

…e now use the equation of dynamic programming in part ii) of Theorem
… prove the following theorem which is the main result of this paper.

…HEOREM 3.1. *Assume that the conditions of Theorem 1.1 or those of Corollary*
…*e satisfied and for all* $k = 0.1, \ldots, N-1$

…f_k *is a continuous function, the functional* h_k *is lower semicontinuous and*
…*d,* U_k *is compact;*

…*for any lower semicontinuous functional* $H_k(x, v, y)$ *on* $X_k \times U_k \times Q_{k+1}$
…*ctional* $\int_{Q_{k+1}} H_k(x, v, y) P_{k+1}(dy, q)$ *is lower semicontinuous on* $X_k \times U_k \times Q_k$.

…*re exists an optimal control* $\widehat{u}$.

…f. It is easily seen that for each $k = 0, 1, \ldots, N-1$, $V_k(x)$ is compact and
…alued map V_k from X_k to U_k defined by

$$V_k(x) := \{v \in U_k : P\{f_k(x, v, q_{k+1}) \in X_{k+1}\} = 1\}$$

…semicontinuous.

We want to prove that if $W_{k+1}(x, q)$ is lower semicontinuous then so are the following functionals:

$$T_k(x, v, q) := \int_{Q_{k+1}} W_{k+1}(f_k(x, v, y), y)\, P_{k+1}(dy, q) + h_k(x, v, q)$$

$$W_k(x, q) := \inf_{v \in V_k(x)} T_k(x, v, q).$$

Indeed, from the hypothesis of the theorem it follows immediately that T_k is lower semicontinuous. Assume that $\{y_n (\in X_k)\} \to y$, $n \to \infty$. Since U_k is compact we may assume, without loss of generality, that for any subsequence $\{y_{n'}\}$ of $\{y_n\}$ there exists a sequence $\{v_{n'} \in V_k(y_{n'})\}$ such that $v_{n'} \to v$, $n' \to \infty$. From the hypothesis i) and the properties of the probability measures it follows that $v \in V_k(y)$.

LEMMA 3.2. *Let Y be a metric space, U a compact metric space, $f : Y \times U \to R$ a lower semicontinuous and lower bounded functional. Assume that there exists a family of closed subsets $\{U_y, y \in Y\}$ of U satisfying the following property*

$$[\forall \{y_n (\in Y)\} \to y,\ \forall \{u_n (\in U_{y_n})\} \to u,\ n \to \infty] \to u \in U_y.$$

Then the functional $\bar{f}(y) := \inf_{u \in U_y} f(y, u)$ is lower semicontinuous and bounded from below.

Indeed, from the hypothesis of the lemma it follows that

$$\forall\, y \in Y\ \exists u_y \in U_y : \bar{f}(y) = f(y, u_y). \tag{3.1}$$

For every sequence $\{y_n (\in Y)\} \to y$, let $\{u_n \in U_{y_n}, n \in \mathbb{N}\}$ be a sequence having the property (3.1) and $\{u_{n'}\}$ a subsequence of $\{u_n\}$ such that $\lim_{n' \to \infty} f(y_{n'}; u_{n'}) = \lim_{n \to \infty} f(y_n; u_n)$. Since U is compact, we may assume that $u_{n'} \to u \in U$, $n' \to \infty$.

It follows from the hypothesis that $u \in U_y$. Hence we have

$$\lim_{n \to \infty} \bar{f}(y_n) = \lim_{n' \to \infty} f(y_{n'}; u_{n'}) \geqslant f(y, u) \geqslant \bar{f}(y),$$

i. e. $\bar{f}$ is lower semicontinuous.

Lemma 3.2 shows that W_k is lower semicontinuous. By induction, it follows readily that the functionals $\{W_k, T_k, k = 0, 1, \ldots, N-1\}$ are lower semicontinuous.

LEMMA 3.3 ([5]). *Let X be a metric space, U a compact metric space, V an upper semicontinuous set-valued map from X to U, $T : X \times U \to R$ a lower semicontinuous and bounded functional. Then the set-valued map G from X to U defined by*

$$G(x) := \{u \in V(x) : T(x, u) = \min_{v \in V(x)} T(x, v)\}$$

is Borelian-closed, i. e for every closed subset E of U the set $\{x \in X : G(x) \cap E \neq \emptyset\}$ is Borelian.

Let us consider the set—valued map G_k from $X_k \times Q_k$ to U_k defined by

$$G_k(x, q) := \{u \in V_k(x) : T_k(x, u, q) = \inf_{v \in V_k(x)} T_k(x, v, q)\}.$$

It is clear that the subset $G_k(x, q)$ is non-empty and closed for all $(x, q) \in \in X_k \times Q_k$. From the lemma 3.3 it follows that set—valued map G_k is Borelian—closed.

From the selection theorem ([6]) it follows that there exists a Borelian function $\widehat{g}_k : X_k \times O_k \to U_k$ such that $\widehat{g}_k(x, q) \in G_k(x, q)\ \forall\ (x, q) \in X_k \times Q_k$ $(k = 0.1, \dots, N - 1)$.

Hence, from the equation of dynamic programming of Theorem 2.1. the control $\widehat{u} = \{\widehat{u}_0 = \widehat{g_0}(\widehat{x}_0),\ \widehat{u}_k = \widehat{g}_k(\widehat{x}_k, q_k),\ k + 1, 2, \dots, N - 1\}$ $(\in \mathcal{U})$ is optimal

Application.

We now apply Theorem 3.1 to the following problem of optimal control for the hydroelectric station « Hoa Binh ».

Let x_n, q_n, W_n, v_n be respectively the volume of the reservoir, the flow of the water arriving to the reservoir, the flow of the water used for the generator and the overflow at the moment n. Further let us denote the period of exploitation by N. Then we have :

$$\begin{cases} x_{n+1} = x_n - w_n - v_n + q_{n+1} \\ x_0 = x_0,\ n = 0,1, \dots, N - 1 \end{cases}$$

with constraints :

$$\underline{w}_n \leqslant w_n \leqslant \overline{w}_n,\ 0 \leqslant v_n \leqslant \overline{v}_n,\ n - 0,1.., N - 1$$

$$P\{X_n \leqslant x_n \leqslant \overline{X}_n,\ Y_n \leqslant h(x_n, w_n, v_n) \leqslant \overline{Y}_n,\ n = 0,1, \dots, N\} = 1$$

where: w_n, $\overline{w}_n$, $\overline{v}_n$, X_n, $\overline{X}_n$, Y_n, $\overline{Y}_n$ are given constants;

$$h(x, w, v) = a[H_t(x) - H_h(w + v)]^b\, w,\ H_t(x) = c\, x^d,$$

$$H_h(y) = e\, y^i,\ a = (176403)^{-1},\ b = 1,\ 10159: c = 2;\ e = 0{,}0294:$$

$d = 0{,}4$; $i = 0{,}63377$.

We have to maximize the total gain functional

$$J(x_0, u) = E\left\{\sum_{n=0}^{N-1} h(x_n, u_n)\right\},\ \text{where } u_n = (w_n, u_n).$$

In actual practice, the conditions stated in Theorem 1.1. are satisfied. The relations between the random variables $\{q_n,\ n = 1, 2, \dots, N\}$ are given by :

$$\widehat{q}_{n+1} = A\,\widehat{q}_n + r_{n+1},\ n = 0,1,\dots, N - 1,$$

where: $\widehat{q}_n = lnq_n - lnq_{n-12}$, A is a constant, r_n has. a normal distribution $N(0, \sigma_n^2)$; q_{-12}, $(q_{-11}, \ldots, q_0)$ are given random variables; $\{r_n, n = 1, 2, \ldots, N\}$ and $(q_{-12}, q_{-11}, \ldots, q_0)$ are independent random variables. Set:

$$q'_n = (q_{n-12}, q_{n-11}, \ldots, q_n),\ n = 1, 2, \ldots, N.$$

We have:

$$x_{n+1} = f(x_n, u_n, q'_{n+1}) := x_n - w_n - v_n + q_{n+1}.$$

To apply Theorem 3.1 it remains to verify its hypothesis ii) or, equivalently, to show that for every $n = 1, 2, \ldots, N-1$, $x > 0$; $y, z \in R_+^{13}$

$$P\{q_{n+1} \langle x / q'_n = z, q'_{n-1} = y\} = P\{q_{n+1} \langle x / q'_n = z\}$$

and the family of the distributions

$$\{F_{n+1}(x, z) := P\{q_{n+1} \langle x / q'_n = z\}, z \in R_+^{13}\}$$

is weakly continuous.

Indeed, for each $z = (z_0, z_1, \ldots, z_{12}) \in R_+^{13}$ we have:

$$P\{q_{n+1} \langle x / q'_n = z\} = P\{\ln q_{n+1} - \ln q_{n-2} \langle \ln x - \ln q_{n-2} / q'_n = z\}$$

$$= P\{A(\ln q_n - \ln q_{n-12}) + r_{n+1} \langle \ln\left(\frac{x}{z_1}\right) / q'_n = z\}$$

$$= P\left\{r_{n+1} \langle \ln\left(\frac{x}{z_1}\right) - A \ln\left(\frac{z_{12}}{z_0}\right)\right\}$$

$$= \Phi\left[\sigma_{n+1}^{-1} (\ln(x/z_1) - A \ln(z_{12}/z_0))\right]$$

where $\Phi(x)$ is the distribution function of the normal law $N(0; 1)$. By an analogous argument, we have:

$$P\left\{q_{n+1} < x / q'_n = z, q'_{n-1} = y\right\} =$$

$$\Phi\left[\sigma_{n+1}^{-1} (\ln(x/z_1) - A \ln(z_{12}/z_0))\right]$$

Hence, from Theorem 3. 1 it follows that there exists an optimal control for the above problem of optimal control for the hydroelectric station « Hoa Binh ».

The author would like to express his thanks to Prof. Nguyen Van Huu for his comments and suggestions.

REFERENCES

[1]. V.I. Arkin and L.I. Krecnetov A *stochastic maximum principle in control problems with discrete-time*. Lecture Note in Math., 1976, V. 550.

[2]. F.I. Beutler and K. W. Ross *Optimal policies for controlled chains with a constraint*. J. Math. Anal. and Appl., 112 (1985), 236—252.

[3]. В. И. Аркини Л. И. Кречетов *Марковские управления в задачах с дискретным, верменем стохастический принцип максимума* – Верояг. Процсссы и упраглецие, М.. « наука » 1978. 8 – 41.

[4]. В.И. Аркин и Л.И. Кречетов *Стохастические множители Лагранжа в задачах управления и экономической динамики* — Вероят проблемы управления в Экономике. М., « наука », 1977, 8 – 40.

[5] П.Г. Губенко и Э. С. Щтатланд *Об управляемых марковских процессах с дискретным времеием* Теория Вероят. и матем. статистика, 7 (1972). 51 – 65.

[6] К. Куратовский *Топология*, т.2, М., « Мир », 1969, 379.

Received September 7, 1988

DEPARTMENT OF MATHEMATICS, HANOI UNIVERSITY — VIET NAM