PROBLEM OF PURSUIT IN LINEAR DISCRETE GAMES WITH STATE INFORMATION

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1. INTRODUCTION

The problem of pursuit in discrete games has been developed in [1—6] under the hypothesis that the pursuer can observe the decisions of the evader. In this paper we shall also be concerned with such a problem, but unlike [1—6] we shall assume that the pursuer observes only the system states.

The organization of the paper is as follows. Section 2 is devoted to the formulation of the problem. In Section 3, a method of construction of a pursuit strategy and some sufficient conditions for pursuit are established. Section 4 gives some estimations of the effect of the proposed method.

2. PRELIMINARIES

Throughout this paper we shall denote by R^m the m- dimensional Euclidean space and by $l_2(R^m)$ the space of all maps $z(.) : Z^+ = \{0, 1, 2, ...\} \to R^m$ with the norm

$$\| z(.) \| := \left(\sum_{k=0}^{\infty} \| z(k) \|^2\right)^{1/2} < +\infty.$$

We shall consider the game (G) described by the difference equations

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) - Cv(k), \; x(0) = x_0, \\ x(k) \in R^n, \; u(k) \in R^p, \; v(k) \in R^q, \\ k = 0, 1, 2, ... \end{cases} \tag{2.1}$$

with the closed convex terminal set $M \subset R^n$, where A, B and C are $n \times n$, $n \times p$ and $n \times q$ matrices. Assume that the pursuit control $u(.)$ and the evasion control $v(.)$ have to satisfy the constraints

$$\| u(.) \| \leqslant \rho, \; \| v(.) \| \leqslant \sigma,$$

(ρ and σ being given positive numbers).

Let l and m be positive integers where $l \geqslant m$. The following information hypothesis for the pursuer will be used in the game (G):

« At any step k, $k \geqslant l$, the pursuer knows only the state $x\left(\left[\frac{k-(l-m)}{l}\right] . l\right)$ of the system ». (2.3)

Here $[\alpha]$ stands for the integer part of α.

Let us define the function $\varphi : Z^+ \to Z^+$ by

$$\varphi(k) = \begin{cases} 0 & \text{if } k \leqslant l, \\ \left[\dfrac{k-(l-m)}{l}\right] . \, l & \text{otherwise.} \end{cases} \tag{2.4}$$

DEFINITION 1. A pursuit strategy for (G) is a map $U : Z^+ \times R^n \to R^p$ such that for any $v(.) \in l_2(R^q)$, $\| v(.) \| \leqslant \sigma$, the map $u(.)$. $Z^+ \to R^p$ defined by the system

$$\begin{cases} x(k+1) = Ax(k) + BU(k, x(\varphi(k))) - Cv(k), \; x(0) = x_0, \\ u(k) = U(k, x(\varphi(k))), \; k = 0, 1, 2, \ldots, \end{cases} \tag{2.5}$$

satisfies the constraint $\| u(.) \| \leqslant \rho$.

DEFINITION 2. Let there be given a positive number ε and a positive integer K. The pursuit process in the game (G) is said to be ε-completable from x_0 after K steps (resp. asymptotically completable from x_0) if there exists a pursuit strategy U such that for any $v(.) \in l_2(R^q)$, $\| v(.) \| \leqslant \sigma$, the trajectory of the system (2.5) associated to U and $v(.)$ satisfies the condition $d(x(k), M) \leqslant \varepsilon$ for some $k \leqslant K$ (resp. $\lim\limits_{k \to +\infty} d(x(k), M) = 0$), where $d(x, M)$ is the distance from x to M.

Let E be the greatest A-invariant subspace satisfying the condition

$$E + M = M . \tag{2.6}$$

Since M is a closed convex set, E is well defined. We denote by R^n/E the factor space of R^n by E and by P the canonical projection of R^n onto R^n/E. Because of the A-invariance of E the induced operator $\bar{A}$ of A on R^n/E is well defined and $P.A = \bar{A}.P$.

3. SUFFICIENT CONDITIONS FOR PURSUIT

3. 1. AUXILLIARY LEMMA.

The result to be presented here will be basic for the construction of a pursuit strategy. We shall assume that

(P_1) : $\mathrm{Im}\, P.A. [C, AC, \ldots, A^{l-1}C] \subset \mathrm{Im}\, P. [B, AB, \ldots, A^{m-1}B]$.

LEMMA 1. *Under the condition (P_1) there exist a map $\tilde{U} : Z^+ \times R^n \to R^p$ and positive constants α, β such that for any $v(.) \in l_2(R^q)$ and the maps $z(.)$, $u(.)$ defined by the systems*

$$\begin{cases} z(k+1) = Az(k) + B\tilde{U}(k, z(\varphi(k))) - Cv(k), \; z(0) = 0, \\ u(k) = \tilde{U}(k, z(\varphi(k))), \; k = 0, 1, 2, \ldots \end{cases}$$

we have

$$\text{i)} \quad \sum_{s=0}^{j+1} \| Pz(s.l) \|^2 \leqslant \beta^2 \sum_{k=0}^{jl-1} \| v(k) \|^2, \tag{3.1}$$

ii) $\sum_{k=0}^{(j+1)l-1} \| u(k) \|^2 \leqslant \alpha^2 . \beta^2 . \sum_{k=0}^{jl-1} \| v(k) \|^2 ,$ (3.2)

for any $j = 0,1,2,...$

Proof. Define the linear maps $F: (R^p)^m \to R^n$ and $G: (R^q)^l \to R^n$ by

$$F(u_0, u_1, ..., u_{m-1}) = \sum_{i=0}^{m-1} A^{m-1-i} Bu_i ,$$

$$G(v_0, v_1, ..., v_{l-1}) = \sum_{i=0}^{l-1} A^{l-1-i} Cv_i . \quad (3.3)$$

Put $W := Im\ P.G$ and $V := Im\ P.\ F$. It is clear from the condition (P_1) that $\overline{A}^l W \subset V$. Hence, for each $\overline{z} \in W$ the optimization problem

$$\begin{cases} \text{Minimize } \| u \| \\ u \in (R^p)^m , P.Fu = \overline{A}^l \overline{z} \end{cases} \quad (3.4)$$

has a unique solution $u^*(\overline{z})$. Furthermore, the solution map $u^* : W \to (R^p)^m$ is linear. Let us define $u(i,.) : R^n / E \to R^p$, $i = 0,1,..., l-1$, and $\widetilde{U} : Z^+ \times R^n \longrightarrow R^p$ by setting

$$(u(0, \overline{z}), u(1, \overline{z}),..., u(l-1, \overline{z})) = \begin{cases} 0 & \text{if } \overline{z} \overline{\in} W \\ (0,0,.., 0,u^*(\overline{z})) & \text{otherwise} \end{cases}$$

and

$$\widetilde{U}(k,z) = u(k - \left[\frac{k}{l}\right] l, P.z), (k,z) \in Z^+ \times R^n \quad (3.5)$$

Now, for fixed $v(.) \in l_2 (R^q)$, consider the trajectory $\{(z(k), u(k)) : k \in Z^+\}$ of the system

$$\begin{cases} z(k+1) = Az(k) + B\widetilde{U}(k,z(\varphi(k))) - Cv(k), z(0) = 0, \\ u(k) = \widetilde{U}(k,z(\varphi(k)), k = 0, 1, 2,... \end{cases} \quad (3.6)$$

For convenience we write

$u_j = (u(jl), u(jl+1),..., u(jl+l-1)),$

$v_j = (v(jl), v(jl+1),..., v(jl+l-1)), j = 0, 1, 2,...$

We shall show by induction that for any $j = 0, 1, 2,...,$

$$u_j = (0, 0,....,0, u^*(P . z(jl))) \quad (3.7)$$

and

$$Pz((j+1)l) = -P . Gv_j . \quad (3.8)$$

Indeed, for $j = 0$, the conditions are clear by the definitions of $\widetilde{U}$ and φ. Assume now that (3.7) and (3.8) hold for an index $j \geqslant 1$. Then $Pz((j+1)l) = -P . Gv_j \in W$. By the definition of φ (see (2.4)) we get

$$u((j+1)l + i) = \widetilde{U}(i, P . z((j+1)l)) = u(i, P.z((j+1)l)),$$

for $l - m \leqslant i < l$. Therefore, $u_{j+1} = (0,0,.., 0,u^* (P.z((j+1)l))$.

On the other hand, using the Cauchy formula for the solution of the equa- (3.6), we obtain

$$z((j+2)\,l) = A^l\, z((j+1)) + \sum_{i=0}^{l-1} A^{l-1-i} \cdot (Bu((j+1)\,l+i) - Cv((j+1)\,l+i))$$

$$= A^l\, z((j+1)\,l) + Fu^*\,(P\,.\,z((j+1)\,l)) - Gv_{j+1}.$$

Hence, from the definition of u^* (see (3.4)) it follows that

$$((j+2)\,l) = -P\,.\,Gv_{j+1}.$$

Now, putting $\alpha := \| u^* \|$, $\beta := \| G \|$ and using (3.7) — (3.8), we can easily y the desired conditions (i), (ii). The proof of the lemma is thus complete.

Remark 1. The above proof shows that $\widetilde{U}$, α and β are found by using only data of the game (G).

.2. FORMULATION OF SUFFICIENT CONDITIONS FOR PURSUIT

Let α and β be the constants defined as in Lemma 1. Our basic hypotheses ormulated as follows.

(P_2) . $\rho > \alpha.\beta.\sigma.$

(C_1) . There exists $w(.) \in l_2(R^p)$, $\|w(.)\| \leqslant \rho - \alpha.\beta.\sigma$, such that the tra- y of the system

$$\begin{cases} y(k+1) = Ay(k) + Bw(k),\ y(0) = x_0 \\ y(k) \in R^n,\ w(k) \in R^p,\ k = 0, 1, 2,\ldots \end{cases} \tag{3.9}$$

es the condition

$$\lim_{k \to +\infty} d(y(k), M) = 0. \tag{3.10}$$

et $\varepsilon > 0$ be fixed.

There exist a positive integer k_0, a positive number $\delta \in (0, \varepsilon)$ and an ele- $w(.) \in l_2(R^p)$, $\| w(.) \| \leqslant \rho - \alpha\beta\sigma$, such that the trajectory of the system ssociated to $w(.)$ satisfies the inequalities

$$d(y(j.\,l), M) \leqslant \delta,\ k_0 \leqslant j \leqslant \frac{K}{l}, \tag{3.11}$$

$$K = (k_0 + [\beta^2\,.\,\sigma^2 / (\varepsilon - \delta)^2])l. \tag{3.12}$$

e are now in a position to formulate the following result.

EOREM 1. *Suppose that the Hypotheses (P_1), (P_2) are satisfied.*

Then

(i) Under Hypothesis (C_2) *the pursuit process in the game* (G) *is* ε*-completable from* x_0 *after* K *steps;*

(ii) Under Hypothesis (C_1) *the pursuit process in the game* (G) *is asymtotically completable from* x_0.

Before proving Theorem 1 we consider a simple example.

Example. Consider the game (G) where

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and $M = \{0\}$. Let $m = 2$ and $l = 3$ be the given parameters in the information hypothesis for (G). Then (P_1) holds. If $\rho - \sigma$ is sufficiently large, then (P_2), (C_1) and (C_2) are satisfied for an arbitrary initial state x_0 (see [6, Theorem 1]).

3.3. PROOF OF THEOREM 1

Constructing a pursuit strategy. Let the game (G) satisfy (P_1), (P_2). We define the map $\widetilde{U} : Z^+ * R^n \to R^p$ as in Lemma 1. Fix $w(.) \in l_2(R^p), \| w(.) \| \leqslant \rho - \alpha\beta\sigma$, and let $\{ y(k) : k \in Z^+ \}$ be the trajectory of the system

$$y(k+1) = Ay(k) + Bw(k),\ y(0) = x_0 \tag{3.13}$$

Consider the map $U : Z^+ \times R^n \to R^p$ defined by

$$U(k, x) := w(k) + \widetilde{U}(k, x - y(\varphi(k))). \tag{3.14}$$

We are going to show that U is a pursuit strategy for (G). Suppose that $v(.) \in l_2(R^q)$, $\| v(.) \| \leqslant \sigma$. Let us consider the system

$$\begin{cases} x(k+1) = Ax(k) + BU(k, x(\varphi(k))) - Cv(k),\ x(0) = x_0, \\ u(k) = U(k, x(\varphi(k))),\ k = 0, 1, 2, \ldots \end{cases} \tag{3.15}$$

Put $z(k) = x(k) - y(k)$ and $\widetilde{u}(k) = u(k) - w(k)$ for $k = 0, 1, 2, \ldots$.
Then it follows from (3.13) — (3.15) that

$$\begin{cases} z(k+1) = Az(k) + B\widetilde{U}(k, z(\varphi(k))) - Cv(k),\ z(0) = 0 \\ \widetilde{u}(k) = \widetilde{U}(k, z(\varphi(k))),\ k = 0, 1, 2, \ldots, \end{cases} \tag{3.16}$$

and the conditions (i), (ii) in Lemma 1 hold for $z(.)$, $\widetilde{u}(.)$ and $v(.)$. Therefore,

$$\| u(.) \| = \| (w + \widetilde{u})(.) \| \leqslant \| w(.) \| + \| \widetilde{u}(.) \| \leqslant \rho - \alpha\beta\sigma + \alpha\beta\sigma \leqslant \rho .$$

Thus, U is a pursuit strategy for (G).

Furthermore, we obtain the following important estimation

$$d(x(k), M) \leqslant d(y(k), M) + \| Pz(k) \|, \tag{3.17}$$

which follows from the definition of the space E (see (2.6)).

Proof of Part (i). Suppose that (P_1), (P_2) and (C_2) are satisfied. Let $w(.) \in l_2(R^p)$ be the function appeared in the statement of (C_2) and U be the corresponding pursuit strategy constructed by the above approach. We shall show that the pursuit process in the game (G) is ε-completable from x_0 after K steps by using this pursuit strategy U where K is as in Hypothesis (C_2).

Suppose that $v(.) \in l_2(R^q)$, $\| v(.) \| \leqslant \sigma$. Let us consider the systems (3.15) and (3.16). In view of Lemma 1 we have

$$\sum_{j=k_0}^{K/l} d^2(x(jl) - y(jl), E) = \sum_{j=k_0}^{K/l} d^2(z(j), E)$$

$$\leqslant \beta^2 . \sum_{k=0}^{K-1} \| v(k) \|^2$$

$$\leqslant \beta^2 . \| v(.) \|^2$$

$$\leqslant \beta^2 . \sigma^2,$$

where k_0 is as in Hypothesis (C_2).

Hence, there exists a positive integer j^*, $k_0 \leqslant j^* \leqslant K/l$, such that

$$d^2(x(j^*l) - y(j^*l), E) \leqslant \beta^2 . \sigma^2 /(Kl - k_0).$$

Since $K = (k_0 + [\beta^2 \sigma^2 / (\varepsilon - \delta)^2])\, l$, we get

$$K/l - k_0 = \beta^2 \sigma^2 / (\varepsilon - \delta)^2,$$

from which it follows that

$$d(x(j^*l) - y(j^*l), E) \leqslant \varepsilon - \delta. \qquad (3.18)$$

On the other hand, since $k_0 \leqslant j^* \leqslant K/l$ and by Hypothesis (C_2) we have

$$d(y(j^*l), M) \leqslant \delta. \qquad (3.19)$$

Put $k^* = j^*l$. Then $k^* \leqslant K$ and by (3.17)—(3.19) we get

$$d(x(k^*), M) \leqslant \varepsilon$$

showing that the pursuit process in the game (G) is ε-completable from x_0 after K steps.

The Part (ii) can be established by an argument analogous to that used in the proof of the Part (i).

4. ON THE EFFECT OF THE PURSUIT

In this section we assume that $0 \in M$. Then the space E defined in Section 2 is contained in M. We shall deal with the set of all initial states x_o from which the pursuit process in (G) can be completed by using our method of pursuit and we shall discuss a relationship between the matrices of the game (G) and the effect of the pursuit.

3.1. THE SET OF THE STATES FROM WHICH THE PURSUIT IS COMPLETED.

We introduce the following notations:

— $X_{stab} := \{ x \in R^n : \lim_{k \to +\infty} A^k x = 0 \}$;

— $\langle A \mid B\rangle := \operatorname{Im}[B, AB, \ldots, A^{n-1}B]$;

— $\langle A \mid B\rangle^0$: the greatest A-invariant subspace of $\langle A \mid B\rangle$ such that the eigenvalues μ of the restriction of A to $\langle A \mid B\rangle^0$ satisfy the condition $|\mu| \leqslant 1$.

THEOREM 2. *Suppose that the game (G) satisfies* (P_1), (P_2) *and the following condition(*):*

(*) *The adjoint matrix A^* of A has no eigenvector which corresponds to an eigenvalue μ with $|\mu| \geqslant 1$ and which is orthorgonal to $E + \langle A \mid B\rangle$.*

Then there exists a convex neighbourhood W of $X_{stab} + E + \langle A \mid B\rangle^0$ such that.

i) *The pursuit process in the game (G) is asymptotically completable from every point $x_0 \in W$;*

ii) *For $\varepsilon > 0$ and each $x_0 \in W$ the pursuit process in the game (G) is ε-completable from x_0 after a finite number of steps.*

Proof. Let us consider the control system

$$\begin{cases} y(k+1) = Ay(k), + Bw(k), \; y(0) = x_0, \; k \in Z+, \\ \| w(\cdot) \| \leqslant \rho - \alpha\beta\sigma. \end{cases} \tag{4.1}$$

Denote by W the set of all states x_0 which can be steered to $X_{stab} + E$ by using this system. Note that $X_{stab} + E$ is an A-invariant space. The condition(*) implies that $X_{stab} + E + \langle A \mid B\rangle = R^n$. Then by a method analogous to that used in the proof of Theorem 1 in [6] we can prove that W is a convex neighbourhood of $X_{stab} + E + \langle A \mid B\rangle^0$.

Now, let $\varepsilon > 0$ and $x_0 \in W$. We have to show that the hypotheses (C_1), (C_2) are satisfied. Then the assertions of Theorem 2 are immediately obtained from Theorem 1.

Let $w(k)$, $k = 0, 1, \ldots, s-1$, be controls of (4.1) steering x_0 to $y(s) \in X_{stab} + E$, that is $y(s) = a + b$ where $a \in X_{stab}$ and $b \in E$. Put $w(k) = 0$ for $k \geqslant s$. Then $w(.) \in l_2(R^p)$, $\| w(.) \| \leqslant \rho - \alpha.\ \beta.\ \delta$. For the trajectory of (4.1) associated to $w(.)$ we have

$$\begin{aligned} d(y(k), M) &\leqslant d(A^{k-s}a, M) + d(A^{k-s}b, M) \\ &\leqslant \| A^{k-s}a \| + d(A^{k-s}b, E) \\ &\leqslant \| A^{k-s}a \| \end{aligned}$$

for all $k \geqslant s$. Since $a \in X_{stab}$, $\lim\limits_{k \to +\infty} d(y(k), M) = 0$. In particular, for any $\delta > 0$ there exists k^* such that $d(y(k), M) \leqslant \delta$ for all $k \geqslant k^*$. Thus, the hypotheses (C_1) and (C_2) are satisfied.

THEOREM 3. *Suppose that the game (G) satisfies the assumptions of Theorem 2 and the following condition*

(**): *The adjoint matrix A^* of A has no eigenvector which corresponds to an eigenvalue μ with $|\mu| > 1$ and which is orthorgonal to E.*
Then the pursuit process in the game (G) is asymptotically completable and ε-completable from every point $x_0 \in R^n$ after a finite number of steps.

Proof. One can easily verify that under conditions (*) and (**) $R^n = X_{stab} + E + \langle A \mid B \rangle^0$. Then the proof is immediate from Theorem 2.

4.2 FURTHER ESTIMATIONS.

We consider the following properties for the given matrices A, B and C of the game (G):

($\mathcal{P}_1$): For each $\sigma > 0$, $R > 0$ there exists $\rho > 0$ such that the pursuit process in the game (G) is asymptotically completable from every point $x_0 \in R^n$, $\| x_0 \| \leqslant R$.

($\mathcal{P}_2$): For each $\sigma > 0$, $R > 0$ and $\varepsilon > 0$ there exist $\rho > 0$ and a positive integer K such that the pursuit process in the game (G) is ε-completable from any $x_0 \in R^n$, $\| x_0 \| \leqslant R$, after K steps.

THEOREM 4. *Assume $M = \{0\}$. Suppose that (A,B,C) satisfies Hypothesis (P_1) and $X_{stab} + < A \mid B > = R^n$. Then the properties $(\mathcal{P}_1)$, $(\mathcal{P}_2)$ hold for (A,B,C).*

COROLLARY 1. *Assume $m \geqslant n$. Then the properties $(\mathcal{P}_1)$, $(\mathcal{P}_2)$ hold for almost* (A, B, C).

Proof. When $m \geqslant n$ the Kalman condition saying that

$$\text{Rank}\,[B, AB, \ldots, A^{n-1}B] = n$$

holds on a dense and open subset of the affine space of structural matrices (A, B, C). Under the Kalman condition we have $\langle A|B \rangle = R^n$, and hence (P_1) holds.

Proof of Theorem 4. Let σ and R be given. For each $\rho > \alpha\beta\sigma$ we put $\Delta(\rho) = \rho - \alpha\beta\sigma$ and consider the control system

$$\begin{cases} y(k+1) = Ay(k) + Bw(k), \; y(0) = x_0, \\ \| w(.) \| \leqslant \Delta(\rho). \end{cases} \tag{4.2}$$

Denote by $W(\rho)$ the set of all states x_0 which can be steered to X_{stab} by using the system (4. 2). We can show as in the proof of Theorem 2 that $W(\rho)$ is a convex neighbourhood of X_{stab}. Put

$$R(\rho) = \sup\{r > 0 : B(0, r) \subset W(\rho)\}$$

where $B(0, r) := \{x \in R^n : \| x \| < r\}$. Clearly, $R(\rho) > 0$. Using the definition of $W(\rho)$, one can easily verify that $\lim_{\rho \to \infty} R(\rho) = +\infty$. This implies that $(\mathcal{P}_1)$ holds for (A, B, C).

Now, let $\varepsilon > 0$ be given. By the definition of X_{stab} the restriction of A to X_{stab} has only eigenvalues μ with $|\mu| < 1$. Hence, there exist $c > 0$ and $s \in (0, 1)$ such that

$$\| A^k x \| \leqslant c \,.\, s^{k+1} \,.\, \| x \|, \; k \in Z^+, \; x \in X_{stab}. \tag{4.3}$$

Let us take a number $\delta \in (0, \varepsilon)$ and put

$$V = \text{convex} \bigcup_{k=0}^{\infty} A^k(B(0, \ \delta c^{-1}) \cap X_{\text{stab}}).$$

Then

$$0 \in V \subset X_{\text{stab}}, \ AV \subset V \text{ and}$$
$$\| A^k x \| \leqslant \delta \text{ for } x \in V \text{ and } k \in Z+. \tag{4.4}$$

For $i = 0,1,2 \ldots$, we denote by $W^i(\rho)$ the set of all states x_0 which can be steered to V after i steps by using the system (4.2). It follows from (4.4) that the sets $W^i(\rho)$ are convex, $0 \in W^i(\rho)$ and $W^i(\rho) \subset W^{i+1}(\rho)$. Furthermore, since V is a neighbourhood of the origin in X_{stab}, by definition $W(\rho) \equiv \bigcup_{i=0}^{\infty} W^i(\rho)$.

Then, in view of Lemma 1 in [7] there exists j^* such that $0 \in \text{int } W^{j^*l}(\rho)$. Putting $r(\rho) = \sup \ \{r > 0 : B(0, \ r) \subset W^{j^*l}(\rho)\}$, we have

$$\lim_{\rho \to +\infty} r(\rho) = + \infty. \tag{4.5}$$

On the other hand, suppose that $x_0 \in W^{j^*l}(\rho)$ and $w(k)$, $k = 0,1,\ldots, j^*l$-1, are controls of (4.2) steering x_0 to V after j^* steps. Putting $w(k) = 0$ for $k \geqslant j^*l$, we obtain $w(.) \in l_2(R^p)$ with $\| w(.) \| \leqslant \Delta(\rho)$. Let $\{ y(k) : k \in Z+ \}$ be the trajectory of (4.2) associated to $w(.)$. Then $y(k) \in V$ for all $k \geqslant j^*l$, and hence by (4.4)

$$\| y(k) \| \leqslant \delta \text{ for all } k \geqslant j^*l.$$

Consequently, Hypothesis (C_2) holds with

$$K \quad (j^* + [\beta^2 \sigma^2 / (\varepsilon - \delta)^2]) \, l.$$

It follows from Theorem 1 that the pursuit process in (G) is ε-completable from every point $x_0 \in W^{j^*l}(\rho)$ after K steps. Combining this fact and (4.5) shows that $(\mathcal{P}_2)$ is satisfied for (A, B, C). The proof is thus complete.

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