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GAUSSIAN RANDOM OPERATORS IN BANACH SPACES (*)

DANG HUNG THANG

I. INTRODUCTION

Let X and Y be separable Banach spaces. By a random operator from X into Y we mean a linear continuous operator from X into $L_0(Y)$ where $L_0(Y)$ stands for the set of all Y-valued random variables. For the motivation of the notion of random operator see our recent paper [8] in which the characteristic function, the convergence and the decomposability of random operators have been studied.

This paper which is a continuation of [8] is devoted to the study of Gaussian random operators in Banach spaces. In Section 2 we introduce the definition of covariance operator of Gaussian random operators. This definition extends the notion of covariance operator of Gaussian cylindrical random variables, see [2], [3]. Theorem 2.4 gives the necessary and sufficient condition for an operator to be the covariance, operator of some Gaussian random operator. We focus on the problem of π_p-decomposability $(0 < p \leqslant \infty)$ of Gaussian random operators in Section 3. We present conditions for π_p-decomposability of a Gaussian random operator in terms of its covariance operator, which may be considered as an extension of S. A. Chobanian, V. I. Tarieladzes results [1] for Gaussian cylindrical measures.

II. COVARIANCE OPERATOR OF GAUSSIAN RANDOM OPERATORS

Fix a probability space $(\Omega, \mathfrak{F}, P)$. Let X and Y be two separable Banach spaces with the duals X' and Y', respectively. The set of all Y-valued random variables (Y-valued $r.v.$'s) is denoted by $L_0(Y)$ and is equipped with the topo-

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logy of convergence in probability. By $L_p(Y)$ ($0 < p < \infty$) we denote the space of Y-valued $r.\ v.$'s for which $E\|x\|^p < \infty$. When $Y = R$ we write L_p instead of $L_p(R)$

A linear continuous operator A from X into L_0 (Y) is called a random operator from X into Y. For some general properties of random operators, see [8].

DEFINITION 2. 1. A random operator A from X into Y is called a *Gaussian random operator* if for each $n \in N$ and $x_1, x_2, \dots, x_n$ in X, the Joint distribution of $Ax_1, Ax_2, \dots, A_n$ is Gaussian. Equivalently, a random operator A is Gaussian if and only if the stochastic process (Ax, y) on $X \times Y'$ is Gaussian.

EXAMPLE 1. Let (T, Σ, m) be a finite measurable space. By a Gaussia random measure W on (T, Σ, m) we mean an independently scattered σ-additive setfunction $W\ \Sigma : \to L_0$ such that, for each A from Σ, $W(A)$ has a Gaussian distribution with mean 0 and variance $m(A)$. Let Y be a Banach space of type 2. It is known [4] that for each $f \in L_2(T, \Sigma, m; Y)$ the random integral $\int f dw$ is defined. Then a random mapping A from $L_2(T, \Sigma, m; Y)$ into Y given by

$$Af = \int f dw$$

is a Gaussian random operator.

PROPOSITION 2. 2. *Let A be a Gaussian random operator from X into Y. Then there exists an unique linear continuous operator M from X into Y such that*

$$(Mx, y) = E(Ax, y)$$

for each $x \in y, y \in Y'$.

Proof. For each $x \in X$ Ax is an Y-valued Gaussian $r. v.$ so $Ax \in L_1(Y)$ Hence A may be considered as a linear mapping from X into $L_1(Y)$. By using the closed graph theorem it is easy to see that A is continuous. Put

$Mx = E\,Ax = \int Ax\ (\omega)\ P(d\omega)$, M is a linear mapping from X into Y. Moreover

$$\sup_{\|x\| \leqslant 1} \|Mx\| = \sup_{\|x\| \leqslant 1} \|E\,Ax\| \leqslant \sup_{\|x\| \leqslant 1} E\|Ax\| < \infty$$

which shows the continuity of M. By the property of Bochner integral we obtain

$$(Mx, y) = (E\,Ax, y) = E\,(Ax, y)$$

The Proposition is proved.

The operator M is called the expectation operator of A. Without loss of generality, from now on we shall always suppose that M is identical to zero.

Let $X \otimes Y'$ be the tensor product of X and Y'. We turn $X \otimes Y'$ into a normed space by considering in it the projective norm defined by

$$\|\ominus\| = \inf \Sigma \|x_i\| \|y_i\|$$

where the infimum is taken over all finite sets of pairs (x_i, y_i) such that $\Theta = \Sigma x_i \otimes y_i$

The role of the tensor product is emphasized by the fact that it enables us to replace a bilinear mapping on $X \times Y'$ by a linear mapping defined on the tensor product $X \otimes Y'$. For more information about the tensor product, we refer to [7].

Let $L(X, Y)$ denote the space of all linear continuous operators from X into Y equipped with the operator norm. For each $u \in L(X, Y)$ we define the following bilinear form on $X \times Y'$ $\quad u(x, y) = (ux, y)$.
By the property of the tensor product, u determines a unique linear mapping $u : X \otimes Y' \to R$ such that

$$u(x \otimes y) = (ux, y)$$

From now on, we shall denote by $\langle u, \theta \rangle$ the value of u at the point $\theta \in X \otimes Y'$. Note that we have the inequality

$$|\langle u, \theta \rangle| \leqslant \|u\| \, \|\theta\| \tag{2—1}$$

Indeed, let $\theta = \Sigma x_i \otimes y_i$ be an arbitrary representation of θ.

Then

$$|\langle u, \theta \rangle| = |\Sigma \langle u, x_i \otimes y_i \rangle| \leqslant \Sigma |\langle u, x_i \otimes y_i \rangle| = \Sigma |(xu_i \, y_i)| \leqslant$$

$$\Sigma \|u\| \, \|x_i\| \, \|y_i\| = \|u\| \, \Sigma \|x_i\| \, \|y_i\|$$

From this, the inequality (2—1) follows.

THEOREM 2. 3. *Let A be a Gaussian random operator from X into Y. Then there exists a unique linear continuous operator R from $X \otimes Y'$ into $L(X, Y)$ such that*

$$\langle R(x \otimes y), s \otimes v \rangle = Cov \{(Ax, y), (As, v)\}$$

for all pairs (x, y) and (s, v) in $X \times Y'$

The operator R is called *the covariance operator of A.*

Proof. Let H_A denote the closed subspace of L_2 spanned by Gaussian *r. v*'s (Ax, y). Consider the bilinear mapping T from $X \times Y'$ into H_A given by

$$T(x, y) = (Ax, y)$$

By the property of the tensor product T determines a unique linear mapping T from $X \otimes Y'$ into H_A such that

$$T(x \otimes y) = (Ax, y)$$

Now we shall show that T is continuous. We have

$$\| T(x \otimes y) \|^2 = \int | Ax, y) \|^2 P(d\omega) \leqslant \| y \|^2 \int \| Ax(\omega) \|^2 P(d\omega).$$

Because of $Ax \in L_2(Y)$ for each $x \in X$, A may be seen as a linear mapping from X into $L_2(Y)$. By using the closed graph theorem we find that A is continuous. Therefore, there exists a constant C such that

$$\int \| Ax(\omega) \|^2 P(d\omega) \leqslant C^2 \| x \|^2. \tag{2—2}$$

Thus, we obtain

$$\| T(x \otimes y) \| \leqslant C \| x \| \| y \|.$$

Let $\theta = \Sigma x_i \otimes y_i$ be an arbitrary representation of θ. Then $\| T(\theta) \| = \| \Sigma T(x_i \otimes y_i) \| \leqslant \Sigma \| T(x_i \otimes y_i) \| \leqslant C \Sigma \| x_i \| \| y_i \|$.

From this we get $\| T(\theta) \| \leqslant C \| \theta \|$ which shows the continuity of T.
Next for each $h \in H_A$ we consider the mapping $Uh : X \to Y$ given by

$$Uh(x) = \int h(\omega)\, Ax(\omega)\, P(d\omega) \tag{2—3}$$

Here the Bochner integral (2–3) exists since

$$\int \| h(\omega)\ Ax(\omega) \| P(d\omega) \leqslant [\int | h^2(\omega) | P(d\omega)]^{1/2} [\int \| Ax(\omega) \|^2 P(d\omega)]^{1/2} < \infty. \tag{2—4}$$

Clearly, Uh is linear. From (2—2) and (2—4) we obtain

$$\| Uh(x) \| = \| E\ hAx \| \leqslant E \| hAx \| \leqslant C \| h \| \| x \| \tag{2—5}$$

which shows that Uh is continuous i. e. $Uh \in L(X, Y)$.
Clearly, the mapping $U : H_A \to L(X, Y)$
$h \to Uh$

is linear. In view of (2—5), we have

$$\| Uh \| = \sup_{\| x \| \leqslant 1} \| Uh(x) \| \leqslant C \| h \|,$$

proving that U is continuous.
Moreover, we have the transposition formula

$$(T\theta, h) = \langle Uh, \theta \rangle \quad \text{for } h \in H_A,\ \theta \in X \otimes Y' \tag{2—6}$$

Indeed, let $\theta = \Sigma x_i \otimes y_i$. Then

$$(T\theta, h) = \Sigma (T(x_i \otimes y_i), h) = \Sigma \int h(\omega)\, (Ax_i(\omega), y_i)\, P(d\omega)$$
$$= \Sigma (\int h(\omega) Ax_i(\omega)\, P(d\omega), y_i) = \Sigma (Uh(x_i), y_i) =$$
$$= \Sigma \langle Uh, x_i \otimes y_i \rangle = \langle Uh, \theta \rangle$$

Hence, U is called the transpose of T and denoted by T^*

Set $R = T^*T$. R is a linear continuous operator from $X \otimes Y'$ into $L(X, Y)$. By (2—6) we have

$$\langle R(x \otimes y), s \otimes v\rangle = \langle T^*T(x \otimes y), s \otimes v\rangle = (T(x \otimes y), T(s \otimes v))$$
$$= E[(Ax, y)(As, v)]$$

The proof is thus complete.

Remark. Denote by $X \widehat{\otimes} Y'$ the completion of $X \otimes Y'$. $X \widehat{\otimes} Y'$ is a Banach space and R can be extended to a linear continuous operator from $X \widehat{\otimes} Y'$ into $L(X, Y)$.

The following theorem gives a criterion for a linear continuous operator $R: X \otimes Y' \to L(X, Y)$ to be the covariance operator of some Gaussian random operator.

THEOREM 2.4 *For a linear continuous operator $R: X \otimes Y' \to L(X, Y)$ to be the covariance operator of some Gaussian random operator, it is necessary and sufficient that*

i) R is positive definite i. e.

$$\langle R\theta, \theta\rangle \geqslant 0 \qquad \text{for all} \quad \theta \in X \otimes Y,$$

and symmetric i. e.

$$\langle R\theta_1, \theta_2\rangle = \langle R\theta_2, \theta_1\rangle$$

ii) For each $x \in X$ the operator $R_x : Y' \to Y$ given by

$$R_x(y) = R(x \otimes y)(x)$$

is the covariance operator of some Y-valued Gaussian r. v.

Proof. Suppose that R is the covariance operator of the Gaussian random operator A. Then

$$\langle R\theta, \theta\rangle = \sum_{i,j} E\{(Ax_i, y_i)(Ax_j, y_j)\} = E\{\sum_i (Ax_i, y_i)\}^2 \geqslant 0,$$

$$\langle R\theta_1, \theta_2\rangle = \sum_j \sum_i E\{(Ax_i, y_i)(Ax_j, y_j)\} = \langle R\theta_2, \theta_1\rangle$$

where $\theta = \Sigma x_i \otimes y_i$, $\theta_1 = \sum_i x_i \otimes y_i$, $\theta_2 = \sum_j x_j \otimes y_j$

It is clear that R_x is the covariance operator of the Y-valued *r. v. Ax*

Conversely, suppose that $R: X \otimes Y' \to L(X, Y)$ is a linear continuous operator satisfying the conditions i) and ii). Consider on $X \otimes Y'$ the function

$$f(\theta) = \exp\{-\langle R\theta, \theta\rangle\}.$$

It is not difficult to check that f satisfies the conditions stated in Theorem 2.3 of [8]. Consequently, there exists a random operator $A: X \to Y$ such that the joint characteristic function of $(Ax_1, y_1), (Ax_2, y_2), \ldots, (Ax_n, y_n)$ is equal to

$$E \exp\{i \Sigma t_k (Ax_k, y_k)\} = f(\Sigma t_k x_k \otimes y_k) =$$
$$= \exp\{-\Sigma\Sigma t_i t_j \langle R(x_i \otimes y_i), x_j \otimes y_j\rangle\}.$$

Thus A is a Gaussian random operator whose covariance operator is precisely R.

COROLLARY 2.5 *Let Y be a Hilbert space. A linear continuous operator $R: X \otimes Y' \to L(X, Y)$ is the covariance operator of some Gaussian random operator $A: X \to Y$ if and only if R is positive definite, symmetric and $\sum_{i=1}^{\infty} \langle R(x \otimes e_i), x \otimes e_i \rangle < \infty$ where (e_i) is the basis of Y.*

PROPOSITION 2.6 *Let $R: X \otimes Y' \to L(X, Y)$ be a covariance operator of a Gaussian random operator. Then there exist a Hilbert space H and a linear continuous operator $T: X \otimes Y' \to H$ such that R can be factorized as follows*

$$\begin{array}{ccc} X \otimes Y' & \xrightarrow{R} & L(X, Y) \\ & T \searrow \quad \nearrow T^* & \\ & H & \end{array}$$

where T^* is the transpose of T in the sense that

$$(T\theta, h) = \langle T^*h, \theta \rangle \qquad h \in H, \theta \in X \otimes Y'$$

and H is minimal (i. e. the image of T is dense in H).
Moreover, the operator T is uniquely (up to an isometry equivalence) defined i. e. if R admits a second factorization $R = T_1^* T_1$ where $T_1: X \otimes Y' \to H_1$ and H_1 is a Hilbert space then there exists an isometry $U: H \to H_1$ such that $T_1 = UT$.

Proof. In proving Theorem 2. 3 we have shown the existence of the desired factorization $R = T^*T$. Suppose that there is another factorization $R = T_1^* T_1$ where $T_1: X \otimes Y' \to H_1$. Let us define the following mapping $U: T(X \otimes Y') \to H_1$ by

$$U(T\theta) = T_1\theta$$

Observe that U is well-defined. Indeed if $T\theta = T\theta'$ then $R(\theta - \theta') = T'T\ (\theta - \theta') = 0$ which implies $\langle T_1^* T_1 (\theta - \theta'), \theta - \theta' \rangle = \| T_1(\theta) - T_1(\theta') \|^2 = 0$ i. e. $T_1\theta = T_1\theta'$. We have $\| UT\theta \|^2 = \| T_1\theta \|^2 = \langle R\theta, \theta \rangle = \| T\theta \|^2$
This means that U is an isometry of $T(X \otimes Y')$ into H_1. U is extended by continuity to $H = \overline{T(X \otimes Y')}$ and we have $T_1 = UT$.

The operator T in the factorization $R = T^*T$ is denoted by $\sqrt{R}$ and called the square root of R.

DEFINITION 2.7 A Gaussian random operator A is said to be *separable* if the Hilbert space $H_A \subset L_2$ spanned by Gaussian *r. v'. s* (Ax, y) is separable.

It would be interesting to know when A is separable.

PROPOSITION 2.8 *A necessary and sufficient condition for A to be separable is that the image $R(X \otimes Y')$ is separable.*

Proof The necessity is clear. Conversely, let $R(X \otimes Y')$ be separable. It is sufficient to prove that the image $T(S) \subset H$ of the unit ball S in $X \otimes Y'$ is separable. Let (y_k) be a sequence in S such that (Ry_k) is dense in $R(S)$. We shall prove that (Ty_k) is dense in $T(S)$. Let $h \in T(S)$. Then there exists an element $y \in S$ such that $h = Ty$. Choosing a subsequence $(y_{n'}) \subset S$ such that $(Ry_{n'})$ converles to Ry in $L(X, Y)$ we have

$\| h - Ty_{n'} \|^2 = \| T(y - y_{n'}) \|^2 = \langle R\ (y - y_{n'}), y - y_{n'} \rangle \leqslant$
$\| Ry - Ry_{n'} \| \ \| y - y_{n'} \| \to 0$, which completes the proof.

THEOREM 2.9. *The random mapping* $A : X \to Y$ *is a Gaussian random separable operator if and only if* A *can be represented in the form*

$$Ax(\omega) = \Sigma\, \gamma_n(\omega)\, B_n x \tag{2-7}$$

where (B_n) *is a sequence in* $L(X, Y)$, (γ_n) *is a sequence of real-valued independent standard Gaussian r. v.' s. The series (2-7) is a.s. convergent in Y.*

This representation of A is called the spectral decomposition of A. The sequence (B_n) is called the spectrum of A.

Proof Let A be a Gaussian separable random operator with the covariance operator R. We have the factorization $R = T^*T$ where $T : X \otimes Y' \to H_A$, H_A is separable. Take an orthogonal basis in H_A

$$e_n = \gamma_n(\omega) \qquad n = 1, 2, \ldots$$

representing a sequence of real-valued independent standard Gaussian *r. v'. s* Put $B_n = T^* e_n \in L(X, Y)$. For each $x \in X$, $y \in Y'$ we have

$$(Ax, y) = T(x \otimes y) = \Sigma\, (T(x \otimes y), e_n)\, e_n =$$
$$\Sigma \langle T^* e_n, x \otimes y \rangle e_n = \Sigma\, (B_n x, y)\, \gamma_n,$$

where the series converges in L_2 hence in distribution. Thus

$Ax(\omega) = \Sigma\, \gamma_n(\omega)\, B_n x$ for almost erery ω by Ito-Nisio's Theorem.

Conversely, if (B_n) is a sequence in $L(X, Y)$ and (γ_n) is a sequence of real-valued independent Gaussian *r. v.* 's such that for each $x \in X$ the series $\Sigma\, \gamma_n B_n x$ is *a.s.* convergent in Y then by using the Banach-Steinhaus Theorem for random operators ([8]), it is easy to see that the random mapping $A : X \to Y$ given by.

$$Ax = \Sigma\, \gamma_n B_n x$$

is a Gaussian separable random operator.

Remark. If N_x is the set of all ω such that the series (2-7) does not converge to $Ax(\omega)$, then the set on which the convergence fails for at least one $x \in X$ is $N = \bigcup_{x \in X} N_x$, an uncountable union of sets of probability *0*, and therefore not necessarily of probability *0*. As we shall see later (Proposition 3-4) the assertion that for ω outside a set of probability *0* the series (2-7) converges to $Ax(\omega)$ for all $x \in X$ holds if and only if A is decomposable.

III. π_p — DECOMPOSABILITY OF GAUSSIAN RANDOM OPERATORS

Recall that a linear operator $u : X \to Y$ is said to be *p-summing* $(o < p < \infty)$ if there exists a constant C such that

$$\Sigma \| ux_n \|^p \leqslant C^p \sup_{\| x' \| \leqslant 1} \{ \Sigma | (x_n, x') |^p \} \tag{3-1}$$

for any finite sequence (x_n) in X. Alternatively, u is p-summing if and only if

$\Sigma \| ux_n \|^p < \infty$ for each sequence $(x_n) \subset X$ such that $\Sigma |(x_n, x')|^p < \infty$ for all $x' \in X'$.

The minimal C for which the inequality (3-1) holds is denoted by $\pi_p(u)$. The class of all *p-summing operators* from X into Y is denoted by $\pi_p(X, Y)$. $\pi_p(X, Y)$ is a Banach space equipped with the norm $\| u \| = \pi_p(u)$. If $0 < p < q$ then

$$\pi_p(X, Y) \subset \pi_q(X, Y).$$

One often refers to a linear continuous operator u from X into Y as an ∞ — summing operator.

DEFINITION 3 — 1. A random operator A from X into Y is said to be π_p — *decomposable* $(0 < p \leqslant \infty)$ if there exists an $\pi_p(X, Y)$ — valued r.v. B such that $Ax(\omega) = B(\omega)x$ for each $x \in X$ and for almost every ω. Instead of saying that A is π_∞ — decomposable, we say that A is decomposable.

PROPOSITION 3. 2. For each π_p - decomposable Gaussian random operator A, the decomposing random variable B must be Gaussian. To prove the Proposition 3. 2, we need the following

LEMMA 3. 1 *Suppose that E is a Banach space and M is a linear subspace of E' such that, for all* $x \in E$,

$$(x, x') = 0 \text{ for all } x' \in M \text{ implies } x = 0. \qquad (3\text{-}2)$$

Then an E - valued r. v. B is Gaussian if for all $x' \in M$, (B, x') *is Gaussian.*

Proof of Lemma 3. 1. We observe that M is dense in E' for the weak topology $\sigma(E', E)$ on E'. Indeed, suppose in the contrary that $M \neq E'$. When E' is equipped with the weak topology. E can be regarded as the dual of E'. By the Hahn-Banach Theorem there exists $x \in E$, $x \neq 0$, such that $(x, x') = 0$ for all $x' \in M$. In view of (3—2) it follows that $x = 0$. A contradiction. Now let x' be an arbitrary element of E'. We have to show that (B, x') is Gaussian. Because M is dense in E' there exists a sequence (x'_n) in M such that (x, x'_n) converges to (x, x') for all $x \in E$. From this $(B(\omega), x'_n)$ converges to $(B(\omega), x')$ for all ω.

As $(B(\omega), x'_n)$ is Gaussian, $(B(\omega), x')$ is Gaussian.

Proof of proposition 3. 2. It is clear that every tensor $\theta \in X \otimes Y'$ defines a linear continuous form on $\pi_p(X,Y)$, name ly $u \to \langle u, \theta \rangle$. Moreover, $\langle u, \theta \rangle = 0$ for all $\theta \in X \otimes Y'$ implies $u = 0$. On the other hand, for each $\theta = \Sigma x_i \otimes y_i$, $\langle B, \theta \rangle = \Sigma(B x_i, y_i) = \Sigma(Ax_i, y_i)$. Since A is Gaussian, $\langle B, \theta \rangle$ is Gaussian. It then suffices to apply the above lemma.

THEOREM 3. 3 *Let A be a Gaussian separable random operator with the spectral decomposition*

$$Ax = \Sigma \gamma_n B_n x$$

Then A is π_p *— decomposable* $(0 < p \leqslant \infty)$ *if and only if the sequence* (B_n) *belongs to* $\pi_p(X, Y)$ *and the series* $\Sigma \gamma_n B_n$ *is a. s. convergent in* $\pi_p(X,Y)$.

Proof. The necessity: Suppose that A is π_p-decomposable. By definition there exists an $\pi_p(X,Y)$-valued *r. v.* B such that $Ax(\omega) = B(\omega)x$ for each $x \in X$ and for almost every ω By Proposition 3.2 B is Gaussian. Let $\tilde{R}$ be the covariance operator of B. It is known [9] that $\tilde{R}$ has the factorization $\tilde{R} = \tilde{T}^*\tilde{T}$ where $\tilde{T}: \pi_p(X,Y)' \to H$ and H is the closed subspace of L_2 spanned by Gaussian *r.v.s.* (B, u'), $u' \in \pi_p(X,Y)'$. When proving Proposition 3.2 we have seen that H is precisely H_A. The sequence (γ_n) represents an orthogornal basis in H_A. It is known [1] that the series $\Sigma \gamma_n \tilde{T}^*\gamma_n$ is *a.s.* convergent in $\pi_p(X,Y)$. Now our assertion will follow if we show that $B_n = \tilde{T}^*\gamma_n$. Indeed, for each $x \in X$ and $y \in Y'$ we have

$$(\tilde{T}^*\gamma_n(x), y) = \langle \tilde{T}^*\gamma_n, x \otimes y\rangle = (\gamma_n, \tilde{T}(x \otimes y)) = (\gamma_n, \langle B, x \otimes y\rangle)$$
$$(\gamma_n, (Bx, y)) = (\gamma_n, (Ax, y)) = \gamma_n, \Sigma (B_k x, y)\ \gamma_k) = (B_n x, y)$$

Consequently, $B_n = \tilde{T}^*\gamma_n$, as desired.

Conversely, suppose that the series $\Sigma\gamma_n B_n$ is *a. s.* convergent in $\pi_p(X,Y)$. Set $B = \Sigma \gamma_n B_n$. B is an $\pi_p(X,Y)$-valued *r.v.* and for each $x \in X$ we have

$$B(\omega)x = \Sigma \gamma_n(\omega) B_n x \quad \text{for almost every } \omega.$$

So

$$B(\omega)x = Ax(\omega) \quad \text{for almost every } \omega,$$

i. e A. is π_p-decomosable.

PROPOSITION 3. 4 *Let A be a Gaussian separable random operator with the spectral decomposition (2–7). Then A is decomposable if and only if there exists set N of probability O such that if $\omega \notin N$ then the series*

$$\Sigma \gamma_n(\omega) B_n x$$

is convergent in Y for all $x \in X$.

Proof. Suppose that A is decomposable. From Theorem 3.3 it follows that there exists an $L(X,Y)$-valued *r.v.* B and a set of probability O such that if $\omega \notin N$ then

$$B(\omega) = \Sigma \gamma_n(\omega) B \quad \text{in } L(X,Y)$$

Therefore, for all $x \in X$ and $\omega \notin N$ we have

$$B(\omega)x = \Sigma \gamma_n(\omega) B_n x$$

Conversely, for each $\omega \notin N$ we define a mapping $B(\omega): X \to Y$ by

$$B(\omega)x = \Sigma \gamma_n(\omega) B_n x.$$

The Banach-Steinhaus Theorem shows that $B(\omega) \in L(X,Y)$. Thus we have an $L(X,Y)$-valued *r.v.* B such that

$$B(\omega)x = \Sigma\gamma_n(\omega) B_n x \text{ for almost every } \omega$$

So

$$B(\omega)x = Ax(\omega) \text{ for almost every } \omega,$$

i. e. A is decomposable.

The following result is basis

THEOREM 3.5. *Let A and A_1 be two Gaussian separable random operators with the covariance operators R and R_1, respectively. Suppose that for all $\theta \in X \otimes Y'$ $\langle R_1\theta, \theta\rangle \leqslant \langle R\theta,\theta\rangle$ and A is π_p—decomposable ($0 < p \leqslant \infty$)*
Then A_1 is also π_p—decomposable.

We begin with the following

LEMMA. *Let R be he covariance operator of the Gaussian separable random operator A. Then we have*

$$\langle R\,\theta, \theta\rangle = \Sigma \ |\langle B_n, \theta\rangle|^2$$

where (B_n) is the spectral sequence of A.

Proof. Let $T: X\otimes Y' \to H_A$ be the square root of R and (γ_n) —an orthogonal basis in H_n. We have

$$T\theta = \Sigma(T\theta, \gamma_n)\,\gamma_n = \Sigma\,\langle T^*\,\gamma_n, \theta\rangle\,\gamma_n$$

We have seen that (Theorem 2.9) $B_n = T^*\,\gamma_n$. From this we get $\langle R\;\theta, \theta\rangle =$

$$= \|T\theta\|^2 = \Sigma\ |\langle T^*\gamma_n, \theta\rangle|^2 = \Sigma\,|\langle B_n, \theta\rangle|^2.$$

Proof of Theorem 3.5. We shall split the proof into two steps.

Step 1. Suppose that $\langle R_1\theta, \theta\rangle = \langle R\theta, \theta\rangle$ for all $\theta \in X \otimes Y'$ Let (B_n) and (B_n^1) be two spectral sequences of A and A_1, respectively. By the above lemma we have

$$\langle R\theta,\theta\rangle = \Sigma\,|\langle B_n, \theta\rangle|^2 = \langle R_1\theta, \theta\rangle = \Sigma\,|\langle B_n^1, \theta\rangle|^2 \tag{3.3}$$

At first we show that $B_n^1 \in \pi_p(X, Y)$ $(n = 1, 2,\ldots)$ Suppose that A is π_p—decomposable by $\pi_p(X, Y)$ valued r.v. B. Then for each $\theta \in X \otimes Y'$ the r.v. $\langle B. \theta\rangle$ is Gaussian with variance $\langle R\theta, \theta\rangle$. Hence for each $p > 0$ there exists a constant C_p such that

$$|\langle R\,\theta,\theta\rangle|^{p/2} = C_p \int_\Omega |\langle B(\omega), \theta\rangle|^p\,P(d\omega).$$

In view of (3—3) we have

$$|\langle B_n^1\,x, \ y\rangle|^p \leqslant \langle R(x \otimes y), x \otimes y\rangle^{p/2} = C_p \int |(B(\omega)\,x, y)|^p \ P(d\omega) \leqslant$$

$$C_p \ \|y\|^p \int \|B(\omega)x\|^p \ P(d\omega).$$

Hence

$$\|B_n^1 x\|^p \leqslant C_p \ \int \|B(\omega)x\|^p \ P(d\omega)$$

For any finite sequence (x_k) in x we have

$$\Sigma\,\|B_n^1 x_k\|^p \leqslant C_p \int \Sigma\,\|B(\omega)x_k\|^p \ P(d\omega) \leqslant$$

$$C_p \int \|B(\omega)\|^p \sup_{\|x'\|\leqslant 1} \{\Sigma\,|(x_n, x')|^p\}\,P(d\omega) = C_p C \sup_{\|x'\|\leqslant 1} \{\Sigma\,|(x_n, x')|^p\},$$ where

$C = \int \|B(\omega)\|^p \; P(d\omega) < \infty$ (since B is Gaussian). Consequently, B_n is p-summing.

Let $A_1 x(\omega) = \Sigma \gamma_n \; \omega \; {}^1_n x$ be the spectral decomposition of A_1. Consider the series $\Sigma \gamma_n \; (\omega) B^1_n$ in $\pi_p(X, Y)$. For each $\theta \in X \otimes Y'$ we have

$$E \exp \langle \sum^n \gamma_k(\omega) B^1_k, \theta \rangle = E \exp \sum^n \gamma_k(\omega) \langle B^1_k, \theta \rangle = \exp \{ -\frac{1}{2} \sum^n | \langle B^1_k, \theta \rangle |^2 \}$$

converging to $\exp \{ -\frac{1}{2} \sum^\infty | \langle B^1_n, \theta \rangle |^2 \} = \exp \{ -\frac{1}{2} \langle R\theta, \theta \rangle \} = E \exp \{ i \langle B, \theta \rangle \}$.

By using the fact that $X \otimes Y'$ is a total linear subspace of the dual $\pi_p(X, Y)'$ and the same argument as in the proof of the Ito-Nisio theorem we find $\pi_p(X, Y)$ valued $r.v.s$ such that

$$\langle S, \theta \rangle = \Sigma \gamma_n(\omega) \langle B^1_n, \theta \rangle \qquad \text{P—a.s.}$$

for each $\theta \in X \otimes Y'$

In particular, taking $\theta = x \otimes y$ we get

$(S(\omega) x, y) = \Sigma \gamma_n(\omega) (B_n x, y) = (Ax(\omega), y)$ for almost every ω. From this it follows that

$A_1 x(\omega) = S(\omega) x$ for almost every ω, i e. A_1 is π_p—decomposable.

Step 2. Suppose that $\langle R_1 \theta, \theta \rangle \leqslant \langle R\theta, \theta \rangle$ for all θ. Put $R_2 = R - R_1$. By using Theorem 2. 4 it is easy to show that R_2 is a covariance operator of some Gaussian separable random operator, say A_2. By the above lemma we have

$$\langle R_1 \theta, \theta \rangle = \Sigma | \langle B^1_n, \theta \rangle |^2, \; \langle R_2 \theta, \theta \rangle = \Sigma | \langle B^2_n, \theta \rangle |^2$$

where (B^1_n) and (B^2_n) are two spectral sequences of A_1 and A_2, respectively. Setting

$$C_{2n-1} = B^1_n, \; C_{2n} = B^2_n \tag{3 — 4}$$

we have

$$\langle R\theta, \theta \rangle = \langle R_1 \theta, \theta \rangle + \langle R_2 \theta, \theta \rangle = \Sigma | \langle C_n, \theta \rangle |^2 \tag{3 — 5}$$

Now let us consider the spectral decomposition of A

$$Ax = \Sigma \gamma_n(\omega) B_n x$$

and the series

$$\Sigma \gamma_n(\omega) C_n x$$

For each $y \in Y'$ we have

$\mathrm{E} \exp \{ (\sum^n \gamma_k(\omega) C_k x, y) \} = \exp \{ -\frac{1}{2} \sum^n | \langle C_k x, y \rangle |^2 \}$ converging to

$$\exp \{ -\frac{1}{2} \sum^\infty | (C_n x, y) |^2 \} = \exp \{ -\frac{1}{2} \langle R (x \otimes y), x \otimes y \rangle \} = \mathrm{E} \exp \{ i (Ax, y) \}$$

By the Ito — Nisio theorem we conclude that the series $\Sigma\gamma_n(\omega)\, C_n x$ converges *a.s.* in Y for each $x \in X$. Set

$$\widetilde{A}\, x\, (\omega) = \Sigma\gamma_n\, (\omega)\, C_n x\, .$$

$\widetilde{A}$ is a Gaussian separable random operator. By lemma and (3 — 5) we get

$$\langle \widetilde{R}\theta,\, \theta\rangle = \langle R\theta,\, \theta\rangle$$

where $\widetilde{R}$ is the covariance operator of $\widetilde{A}$.

From step 1 it follows that $\widetilde{A}$ is π_p— decomposable. Hence, by Theorem 3. 3 the series $\Sigma\gamma_n\, (\omega)\, C_n$ is *a.s.* convergent in $\pi_p\, (X, Y)$. For any bounded sequence of real numbers (t_k) the series $\Sigma t_k\, \gamma_k\, (\omega)\, C_k$ is also *a.s.* convergent in $\pi_p\, (X, Y)$. If we put

$$t_{2n-1} = 1\ ,\ t_{2n} = 0\ \widetilde{\gamma}_n = \gamma_{2n-1}\ ,$$

then by (3 — 4) the series $\Sigma\widetilde{\gamma}_n\, B_n^1$ is *a.s.* convergent in $\pi_p\, (X, Y)$.

Put

$$\widetilde{A}_1 x(\omega) = \Sigma\, \widetilde{\gamma}_n(\omega)\, B_n' x.$$

$\widetilde{A}_1$ is π_p -decomposable and we have

$$\langle \widetilde{R}_1\theta,\, \theta\rangle = \Sigma\, |\, \langle B_n^1,\, \theta\, \rangle\, |^2 = \langle\, R_1\theta,\, \theta\, \rangle$$

From step 1 it follows that A_1 is π_p - decomposable. The proof of the Theorem is thus complete.

COROLLARY 3.6. *Let A and B be two independent Gaussian separable random operators with the expectation operators zero. If $A + B$ is π_p -decomposable then both A and B are π_p - decomposable.*

In the sequel we shall find conditions on the covariance operator R such that the corresponding Gaussian random operator A in π_p -decomposable.

DEFINITION 3.7 Let Z be a Banach space. A linear operator T from $X \otimes Y'$ into Z is said to be (r, π_p) - *summing* $(0 < r < \infty,\ 0 < p \leqslant \infty)$ if for each sequence (θ_n) in $X \otimes Y'$ such that $\Sigma\, |\, \langle\, u,\, \theta_n\, \rangle\, |\, ^r < \infty$ for all $u \in \pi_p\, (X,Y)$ we have $\Sigma\, \|\, T\, \theta_n\, \|\, ^r < \infty$. Equivalently, T is (r, π_p) - summing if and only if there exists a constant C such that

$$\Sigma\, \|\, T\, \theta_n\, \|\, ^r \leqslant C^r \sup_{\|u\|_{\pi_p} \leqslant 1} \{\, \Sigma\, |\, \langle u,\, \theta_n\rangle\, |\, ^r\}$$

for any finite sequence (θ_n) in $X \otimes Y'$. Because of $\pi_p\, (X,Y) \subset \pi_q\, (X,Y)$ whenever $p < q$, the (r, π_p) - summing operators are (r, π_q) - summing if $p < q$. As we shall see later, the converse is not generally true, unless Y is finite — dimensional.

By the same argument as in the proof of Pietschs theorem we get

THEOREM 3.8 *A continuous linear operator $T : X \otimes Y' \to Z$ is (r, π_p)-summing if and only if there exists a finite measure μ on the unit ball U of $\pi_p(X, Y)$ such that*

$$\| T\theta \|^r \leqslant \int_U | \langle u, \theta \rangle |^r \mu(du)$$

for all $\theta \in X \otimes Y'$

THEOREM 3.9 *Let A be a Gaussian random operator from X into Y with the covariance operator R. If A is π_p-decomposable then the operator $T = \sqrt{R}$ is (r, π_p)-summing for all $r > 0$.*

Proof Suppose that A is π_p-decomposable by an $\pi_p(X,Y)$-valued *r.v.* B. Then for each $\theta \in X \otimes Y'$ the *r.v.* $\langle B, \theta \rangle$ is Gaussian with variance $\langle R\theta, \theta \rangle$. Hence for each $r > 0$ there exists a constant C_r such that

$$\| T\theta \|^r = C_r \int_\Omega | \langle B(\omega), \theta \rangle |^r P(d\omega)$$

For each finite sequence (θ_n) in $X \otimes Y'$ we have

$$\Sigma \| T\theta_n \|^r = C_r \int \Sigma | \langle B(\omega), \theta_n \rangle |^r P(d\omega) =$$

$$C_r \int \| B(\omega) \|^r \Sigma \left| \left\langle \frac{B(\omega)}{\| B(\omega) \|}, \theta_n \right\rangle \right|^r P(d\omega) \leqslant$$

$$C_r \int \| B(\omega) \|^r P(d\omega) \{ \sup_{\| u \| \leqslant 1} \Sigma | \langle u, \theta_n \rangle |^r \} = C_r C \sup_{\| u \| \leqslant 1} \Sigma | \langle u, \theta_n \rangle |^r$$

where $C = \int \| B(\omega) \|^r P(d\omega) < \infty$ (since B is Gaussian).
Thus T is (r, π_p)-summing

THEOREM 3. 10. *Suppose that $\pi_p(X, Y)$ is of type 2. Then a Gaussian separable random operator $A : X \to Y$ is π_p-decomposable if and only if the operator $T = \sqrt{R}$ is $(2, \pi_p)$-summing, where R is the covariance operator of A.*

Proof. By Theorem 3. 9 it remains to prove the «*if*» part. Assume that T is $(2, \pi_p)$-summing. By Theorem 3.8 there exists a finite measure μ on the unit ball U of $\pi_p(X, Y)$ such that

$$\| T\theta \|^2 \leqslant \int_U | \langle u, \theta \rangle |^2 \mu(du). \tag{3-6}$$

Since $\pi_p(X, Y)$ is of type 2 by a result in [4], there exists an $\pi_p(X, Y)$-valued Gaussian *r. v.* B such that

$$E \exp \{ i \langle B, u' \rangle \} = \exp \left\{ -\frac{1}{2} \int_U \left| \langle u, u' \rangle \right|^2 \mu(du) \right\}$$

for all $u' \in \pi_p(X, Y)'$. Taking $\theta = u'$ we get

$$\mathrm{E} \exp \{ i \langle B, \theta \rangle \} = \exp \left\{ -\frac{1}{2} \int_U \left| \langle u, \theta \rangle \right|^2 \mu(du) \right\}. \tag{3-7}$$

Let $\widetilde{A}$ be the Gaussian random operator generated by B, i. e.

$$\widetilde{A}x(\omega) = B(\omega)x.$$

From (3-7) we have

$\langle \widetilde{R}\,\theta, \theta\rangle = \int_U |\langle u, \theta\rangle^2 \mu\ (du)$, where $\widetilde{R}$ is the covariance operator of $\widetilde{A}$.

From (3 — 6) we get $\langle R\,\theta, \theta\rangle \leqslant \langle R\ \theta,\ \theta\rangle$. By Theorem 3. 5 we conclude that A is π_p-decomposable.

THEOREM 3. 11. *Suppose that $\pi_p(X, Y)$ is of cotype 2. Then a Gaussian separable random operator $A : X \to Y$ is π_p-decomposable if and only if the transpose T^* of $T = \sqrt{R}$ is an 2-summing operator from H_A into $\pi_p(X, Y)$, where R is the covariance operator of A.*

Proof Suppose that A is π_p-decomposable. Then there exists an $\pi_p(X,Y)$ — valued Gaussian *r.v.* B such that

$$Ax(\omega) = B(\omega)x \qquad \text{for almost every } \omega$$

Let $\widetilde{R}$: $\pi_p(X,Y) \to \pi_p(X,Y)$ be the covariance operator of B. $\widetilde{R}$ has the factorization $\widetilde{R} = \widetilde{T}^*\widetilde{T}$ where $\widetilde{T}$: $\pi_p(X,Y)' \to H_A$ (see the proof of Theorem 3.3). Since $\pi_p(X,Y)$ is of cotype 2 by the result of [1], the operator $\widetilde{T}^*$: $H_A \to \pi_p(X,Y)$ is 2-summing. Now our assertion will follow if we show that $T^* \equiv \widetilde{T}^*$. Indeed for each $h \in H_A$, $x \in X$ and $y \in Y'$ we have

$$\begin{aligned}(\widetilde{T}^*h\ (x), y) &= \langle \widetilde{T}^*h, x \otimes y\rangle = (h, \widetilde{T}\ (x \otimes y)\ ) = (h, \langle B, x \otimes y\rangle\ )\\ &= (h, (B(\omega)x, y)) = (h, (Ax, y)\ ) = (h, T(x \otimes y\ )) = \langle T^*h, x \otimes y\rangle\\ &= T^*(h(x), y).\end{aligned}$$

Hence $\widetilde{T}^* \equiv T^*$ as desired.

Conversely, assume that T^*: $H_A \to \pi_p(X, Y)$ is 2-summing. In view of the Schwartz Radonification theorem [6] $\mu = T^*(\gamma_2)$ is a Radon measure on $\pi_p(X, Y)$, where γ_2 is the Gaussian cylindrical measure on H_A with the characteristic function $\exp\left\{\frac{-\|h\|^2}{2}\right\}$ The characteristic function $\widehat{\mu}$ is equal to

$$\widehat{\mu}(u') = \exp\left\{-\frac{1}{2}\ \|(T^*)^*u'\|^2\right\} \qquad (u' \in \pi_p(X,Y)')$$

where $(T^*)^*$: $\pi_p(X,Y)' \to H_A$.

For each $h \in H_A$ and $\theta \in X \otimes Y'$ we have

$\langle (T^*)^*\ \theta, h\rangle = \langle \theta, T^*h\rangle = (T\,\theta, h)$

which shows that $(T^*)^*\theta = T\,\theta$. Hence

$$\widehat{\mu}(\theta) = \exp\left\{\frac{-\|T\theta\|^2}{2}\right\}.$$

For each $\theta \in X \otimes Y'$ we have

$$E \exp \langle \sum^{n} \gamma_k(\omega) B_k, \theta \rangle = \exp \left\{ -\frac{1}{2} \sum^{n} |\langle B_k, \theta \rangle|^2 \right\} \text{converging to}$$

$$\exp\left\{ -\frac{1}{2} \sum^{\infty} |\langle B_n, \theta \rangle|^2 \right\} = \exp\left\{ -\frac{1}{2} \| T\theta \|^2 \right\} = \widehat{\mu}(\theta),$$

where (B_n) is the spectral sequence of A.

By the same argument as in proving Step 1 of Theorem 3.5 we conclude that A is. π_p-decomposable.

Remark. Let us observe that the «*if*» part is always true without any additional assumption on $\pi_p(X,Y)$.

Theorems 3.10 and 3.11 lead us to the question of which Banach spaces X and Y have the property that $\pi_p(X,Y)$ is of type 2 or is of cotype 2.

THEOREM 3.12. *Let H be a Hitbert space. Then*

1) $\pi_2(X,H)$ is of type 2 if X is the dual of a Banach space of type 2.

2) $\pi_2(H, Y)$ is of cotype 2 if Y is of cotype 2.

Proof 1) Assume that $X = E'$. Let $\Lambda_2(E', H)$ denote the set of all operators T from E' into H for which the function

$$y \to \exp\{-\| Ty \|^2\}$$

is a cheracteristic function of some E-valued Gaussian *r. v.* X_T. It is known [5] that $\Lambda_2(E' H)$ is a Banach space equipped with the norm.

$$\| T \|^2_{\Lambda_2} = E \| X_T \|^2.$$

The correspondence $T \to X_T$ is an isometry of $\Lambda_2(E', H)$ into $L_2(E)$

When E is of type 2, it is known [1] that $\Lambda_2(E', H) = \pi_2 \ (E', H)$, Hence there exist two constants C_1 and C_2 such that

$$C_2 \| T \|^2_{\Lambda_2} \leqslant \| T \|^2_{\pi_2} \leqslant C_1 \| T \|^2_{\Lambda_2}$$

Let $r_1, r_2, \dots$ be the Rademacher sequence on the probability space ([0, 1], $\mathcal{B}$, dt) and let $T_1, T_2, \dots T_n$ be a finite sequence in $\Lambda_2(E', H)$. Then

$$\int_0^1 \| \sum T_n r_n(t) \|^2_{\lambda_2} dt = \int_0^1 E \| \sum X_{T_n} r_n(t) \|^2 dt =$$

$$E \int_0^1 \| \sum X_{T_n} r_n(t) \|^2 dt \leqslant K E\{ \sum \| X_{T_n} \|^2 \} = K \sum \| T_n \|^2_{\Lambda_2}$$

The last inequality used the fact that E is of type 2.

Hence $\Lambda_2(E', H)$ is of type 2 so is $\pi_2(E', H)$

2) Suppose Y is of cotype 2. Then by the results of [1] $T \in \pi_2(H, Y)$ if and only if $T^* \in \Lambda_2(Y', H)$ and we have the constants C_1, C_2 such that

$$C_2 \| T^* \|^2_{\Lambda_2} \leqslant \| T \|^2_{\pi_2} \leqslant C_1 \| T^* \|^2_{\Lambda_2}.$$

Let $T_1, T_2, \ldots, T_n$ be a finite sequence in $\pi_2(H, Y)$. Then

$$\int_0^1 \| \Sigma T_n r_n(t) \|^2_{\pi_2} dt \geqslant C_2 \int_0^1 \| \Sigma T_n^* r_n(t) \|^2_{\Lambda_2} dt =$$

$$C_2 \int_0^1 E \| \Sigma X_{T_n^*} r_n(t) \|^2 \, dt = C_2 E \int_0^1 \| \Sigma X_{T_n^*} r_n(t) \|^2 dt \geqslant$$

$$C_2 \, C \, E \{ \Sigma \| X_{T_n^*} \|^2 \} = C_2 \, C \, \Sigma \| T_n^* \|^2_{\Lambda_2} \geqslant C_1^{-1} C \, C_2 \, \Sigma \| T_n \|^2_{\pi_2}$$

This proves that $\pi_2(H, Y)$ is of cotype 2.

PROPOSITION. 3. 13 *Let X, Y be two separable Hilbert spaces and $A : X \to Y$ a Gaussian separable random operator. Then the following assertions are equivalent*

1) A is π_p — decomposable for $0 < p < \infty$

2) A is π_2 — decomposable.

3) $\sum_j \sum_i \langle R(e_i \otimes f_j), e_i \otimes f_j \rangle < \infty$

4) $\sum_j \sum_i \| B_j e_i \|^2 < \infty$.

Here R is the covariance operator of A, (B_j) is the spectral sequence of A and (e_i), (f_j) are the basises in X, Y, respectively.

Proof 1) $\leftrightarrow$ 2) is trivial because all the classes $\pi_p(X, Y)$ coincide ($0<p<\infty$).

2) $\to$ 3): By Theorem 3.10 the operator $T = \sqrt{R}$ is $(2, \pi_2)$—summing. For each $u \in \pi_2(X, Y)$ we have

$$\Sigma\Sigma \, | \langle u, e_i \otimes f_j \rangle |^2 = \sum_{i\,j}\Sigma \, | \langle ue_i , f_j \rangle |^2 = \sum_i \| ue_i \|^2 < \infty.$$

So

$$\Sigma\Sigma \langle R(e_i \otimes f_j), e_i \otimes f_j \rangle = \Sigma\Sigma \| T(e_i \otimes f_j) \|^2 \leqslant \infty.$$

3) $\to$ 4): It is not difficult to check that

$$\Sigma\Sigma \| B_j \, e_i \|^2 = \Sigma\Sigma \langle R(e_i \otimes f_j), e_i \otimes f_j \rangle.$$

4) $\to$ 2): We have $\Sigma \| B_n \|^2_{\pi_2} = \sum_{i\,j}\Sigma \| B_j e_i \|^2 < \infty$.

Because $\pi_2(X, Y)$ is a Hilbert space it follows that the series $\Sigma \, \gamma_n \, B_n$ is *a.s.* convergent in $\pi_2(X, Y)$. By Theorem 3.3 A is π_2—decomposable.

Finally, we give an example showing that there exists an (r, π_∞) —summing operator which is not (r, π_p)—summing for $p < \infty$.

Let H be a Hilbert space with the basis (e_n) and (s_n) be a sequence of numbers such that $\sup |s_n| < \infty$.

We define a Gaussian separable random operator A from H into H by means of.

$$Ax(\omega) = \Sigma(x, e_n)\, e_n\, s_n\, \gamma_n(\omega)$$

By Proposition 3.13 A is π_p-decomposable ($p < \infty$) if and only if.

$$\sum_{i,j} \| B_j e_i \|^2 = \Sigma\, |s_n|^2 < \infty. \tag{3—8}$$

On the other hand, as shown in [8], A is decomposable if and only if.

$$\Sigma \exp\left\{-\frac{t}{|s_n|^2}\right\} < \infty \text{ for some } t > 0. \tag{3—9}$$

So if the sequence (s_n) is chosen such that (3—9) holds but (3—8) fails then by Theorem 3.10 and Theorem 3.9 the operator $T = \sqrt{R} : H \otimes H \to L_2$ is (r, π_∞)-summing but not (r, π_p)-summing $(p < \infty)$.

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DEPARTMENT OF MATHEMATICS, HANOI UNIVERSITY, VIET NAM