ACTA MATHEMATICA VIETNAMICA
Tom 6 No.2 (1981)

ON THE DENSITY OF UNIVERSAL PROBABILITY DISTRIBUTIONS ON A FRÉCHET SPACE

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Throughout the paper we shall preserve the terminology and notation in [3]. In particular, by E we shall denote a separable Fréchet space and $\mathcal{P}$ the class of all probability distributions (p. d.) on E endowed with the weak convergence $\Rightarrow$. A p. d. p in $\mathcal{P}$ is said to be *infinitely divisible* (*i. d.*) if for every $n = 1,2, \ldots$ there exists a p. d. q_n such that $p = p_n^{n}$,. where the asterisk denotes the convolution operation and the power is taken in this sense. Let's signe by δ (b) the p. d. concentrated at the point $b \in E$. Given a finite Borel measure μ on E, let e (μ) denote the Poisson p. e. defined by

$$e(\mu) := e^{-\mu(E)} \left(\delta(0) + \frac{\mu}{1!} + \frac{\mu^{*2}}{2!} + \ldots\right)$$

For every Borel measure μ on E and $c > 0$ let $T_c \mu$ denote a Borel measure defined by

$$T_c \mu (B) := \mu (c^{-1} \cdot B) \qquad (B \subset E)$$

A p. d. q is said to belong to the *domain of partial attraction* of a p. d. p if there exist a subsequence $\{n_k\}$ of natural numbers and a sequence $\{r_k\}$ of positive numbers such that the sequence $\left\{T_{r_k} q^{*n_k}\right\}$ is shift convergent to p. Further, a. p. d. q is called *universal* if it belongs to the domain of partial attraction of every i. d. p. d. on E.

The problem of the existence of universal p. d. for i. d. p. d. 's on R^1 was first discussed by Doeblin in his famous paper [2]. An extension of Doeblin Theorem to linear spaces was made by Baran'ska [1] in a Hilbert space, Nguyễn Văn Thu [5] in a Banach space and by the Author in a Fréchet space. The purpose of this paper is to prove the density of universal p. d. 's. Namely we get the following Theorem:

THEOREM. *The class of all universal p. d. 's on E is dense in $\mathcal{P}$. Moreover the class of all i. d. universal p. d. 's is dense in the class of all i. d. p. d. 's on E.*

Proof . Let $\left\{ p_n = \delta(d_n) * e(F_n)\ , n = 1, 2, \ldots \right\}$ be the dense sequence of i. d. p. d. 's as in the proof of Theorem 1 [3]. Let's put

$$G_m := \sum_{n=m}^{\infty} 2^{-n^2} T_{2n^3} F_n\ , m = 1, 2, \ldots$$

Then by the same way as in the proof of Theorem 1 [3] we can show that the p. d. 's $p_m := e(G_m)$, $m = 1, 2, \ldots$, are universal p. d. 's, namely if p is an i. d. and $\{n_k\}$ is the subsequence of natural numbers such that

(1) $$p_{n_k} \Rightarrow p$$

then for all m

(2) $$\delta(d_{n_k})^* \ T2^{-n_k^3} \ (q_m^{*2^{n_k^2}}) \Rightarrow p$$

Moreover, it is easy to check that

(3) $$q_m \Rightarrow \delta(0)$$

On the other hand, if η is any p. d. concentrated on a bounded subset of E then by the same reason as in Theorem 3 [3] we infer that

$$T_{n^-\sqrt{\log_2 n}} (\eta^{*n}) \Rightarrow \delta(0)$$

which together with (2) implies the universality of $\eta * q_m$, i. e.

$$\delta(d_{n_k}) * T_{2^{-n_k^3}} (\eta * q_m)^{*2^{n_k^2}} \Rightarrow p$$

whenever (1) holds. Hence by Theorem II. 6. 3 [6] and by (3) the first assertion of the Theorem is proved.

For the proof of the second assertion of the Theorem we note that for every $a \in E$

$$T_{n^-\sqrt{\log_2 n}} (e(\delta(a)))^{*n} \Rightarrow \delta(0)$$

which implies

$$T_{n^-\sqrt{\log_2 n}} (e(\varphi))^{*n} \Rightarrow \delta(0)$$

if φ is a measure concentrated on a finite subset of E. This together with (2) implies that $e(\varphi)^* q_m$, $m = 1, 2, \ldots$, are i. d. universal p. d. ' s. Further by Lemma 2 [3] and by (3) these p. d. 's are dense in the class of all i. d. p. d.' s on E. Thus the Theorem is fully proved.

REMARK. A similar result for Random Measures and Point Processes was obtained in [4].

Received June 25, 1981.

REFERENCES

[1] J. Baransk, *Universal Doeblin Distributions in à Hilbert Space*, Bull. Acad. Polon. Sci. Ser. Math. Astron. Phys., Vol XXVI, No. 4(1978), 343 - 316.

[2] W. Doeblin, *Sur l'ensemble de puissance d'une loi de probabilité*, Bull. Sci. Math. 6 (1939), 71 — 96.

[3] Hồ Đăng Phúc, *Universal Distribution for Infinitely Divisible Distributions on Fréchet Space*, Ann. Inst. Henri Poincaré, Section B, Vol. XVII, No. 2 (1981), 219 - 227.

[4] Hồ Đăng Phúc, *On Doeblin's Theorem for Random Measures and Point Processes*, Bull. Acad. Polon. Sci. , Se,r. Math. Astron. Phys., 1981, [to appear].

[5] Nguyễn Văn Thu, *A New Version of Doeblin's Theorem*, Ann. Inst. Henri Poincaré, Section B, Vol. XVII No. 2 (1981), 213 - 217.

[6] K. R. Parthasarathy, *Probability Measures on Metric Space*, New York, London, Press, 1967.