ACTA MATHEMATICA VIETNAMICA
TOM 5, N° 1 (1980)

ON THE EXTENSION OF STABLE CYLINDRICAL MEASURES

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I — INTRODUCTION

Let X be a separable Banach space with dual X^*. Let $T \in \mathcal{L}(X^*, L^\alpha)$ $0 < \alpha \leqslant 2$ i.e. T is a linear continuous operator from X^* into L^α. Consider the functional $\lambda : X^* \to R$ defined by

$$\lambda(x^*) = \exp\{-\| T x \|^\alpha\}$$

It is easy to see tha $\lambda(x^*)$ is a characteristic functional (ch. f.) of a cylindrical measare Λ^T on X. The set of all operators $T \in \mathcal{L}(X^*, L^\alpha)$ such that Λ^T can be extended into a Radon measure will be denoted by $\mathcal{C}_\alpha(X^*, L^\alpha)$. The problem considered here is that of determining conditions on T which are necessary and sufficient for T to belong to the class $\mathcal{C}_\alpha(X^*, L^\alpha)$. It is noted that for the case $\alpha = 2$ this problem has been solved by S. A. Chobanjan anh V. I. Tarieladeze [3]: If X is of type 2 then $T \in \mathcal{C}_2(X^*, L^2)$ if and only if T is 2-summing.

In this paper we shall try to solve the above mentioned problem for the case $1 < \alpha < 2$. Theorem 3.1 gives the necessary condition for $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$. This condition is also sufficient when X is a closed subspace of L^p $(1 < p \leqslant 2)$ and not sufficient when $X = L^p$ $(p > 2)$. Next, we shall show that the assertion that $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$ if and only if T is α-summing holds if and only if X can be immersed in *some* L^α and is of stable-type α. In this way we obtain a probabilistic description of these spaces. It is interesting to note that here for the case $\alpha < 2$ there are no analogues of the results in the case $\alpha = 2$.

II — PRELIMINARIES

2.1. Spaces of stable-type α. Let X be a separable Banach spacee. X is said to be of stable-type α $(0 < \alpha \leqslant 2)$ if for each sequenca $(x_n)_{n=1}^{\infty} \subset X$ with the property $\sum_{n=1}^{\infty} \| x_n \|^\alpha \infty$, the series $\sum_{n=1}^{\infty} x_n \theta_n^{(\alpha)}$ is convergent a.s. where $(\theta_n^{(\alpha)})_{n=1}^{\infty}$ are independent identically distributed real-valued random variables with the ch. f. $\exp\{-|t|^\alpha\}$.

A theorem by Maurey, Pisier (1976) [7] gives the following purely geometrical characterization of spaces of stable-type α: X is of stable-type α if and only if l^α is not finitely representable in X. Example: Each Banach space in of stable-type α with $0 < \alpha < 1$. L^p is of stable-type $\alpha\,(1 \leqslant \alpha < 2)$ if and only if $p > \alpha$.

X is said to be stable-cotype α if for each sequence $(x_n)_{n=1}^{\infty} \subset X$ such that the series $\Sigma\, x_n\, \theta_n^{(\alpha)}$ is convergent a.s. in X it follows $\Sigma \|x_n\|^\alpha < \infty$. It is known that each Banach space is of stable-cotype α with $0 < \alpha < 2$ (see [7]).

2.2. p-summing operators Let X, Y be Banach spaces. An operator $T \in \mathcal{L}(X, Y)$ is said to be p-summing ($p > 0$) if for each sequence $(x_n) \subset X_\infty$ such that $\sum_{n=1}^{\infty} |< x_n, x^* >|^p < \infty$ for each $x^* \in X$ we have $\sum_{1=n}^{\infty} \|Tx_n\|^p < \infty$. In other words, T is p-summing if T carries each weakly p-absolutely summable sequence from X into the strongly p-absolutely summable sequence in Y. This definition is equivalent to the following one. There exists a constant $C > 0$ such that for every finite collection $(x_n) \subset X$.

$$(\Sigma \|Tx_n\|^p)^{1/p} \leqslant C \sup_{\|x^*\| \leqslant 1} (\Sigma < x_n; x^\alpha >|^p)^{1/p}.$$

An operator $T \in \mathcal{L}(X, Y)$ is said to be completely summing if T is p-summing for each $p > 0$. We denote the class of all p-summing operators from X into Y by $\pi_p(X, Y)$ and denote the class of all completely summing operators from X into Y by $\pi_o(X, Y)$ If $0 < p < q$ then $\pi_p(X, Y) \subset \pi_q(X, Y)$. For more information about p-summing operators see [9], [10]. The connection between spaces of stable-type α and α-summing operators is established by the following theorem.

2.2.1. *Maurey — Pisier's Theorem* [7] *X is of stable-type α if and only if for every quotient space G of the dual space X^* and for any Banach space Y we have $\pi_\alpha(G, Y) \equiv \pi_o(G, Y)$.*

In particular, if X^* is of stable-type α then $\pi_\alpha(X, Y) \equiv \pi_o(X, Y)$.

2.3. p-Radonnifying operators. Let Λ be a cylindrical (probability) measure on X. The characteristic functional (ch. f.) of a cylindrical measure is defined as follows

$$\widehat{\Lambda}(x^*) = \int_R e^{iu}\, \Lambda_{x^*}(du),$$

where Λ_{x^*} is the measure on R given by

$$\Lambda_{x^*}(B) = \Lambda\{x : < x, x^* > \in B\}.$$

Λ is said to be of type p if

$$\sup_{\|x^*\| \leqslant 1} \left[\int_R |u|^p \Lambda_{x^*}(du) \right]^{1/p} < \infty.$$

We say that Λ is a Radon measure if Λ can be extended into a Radon measure on X. Then its extension is also denoted by Λ.

A Radon measura μ is said to be of order p if

$$\int_X \|x\|^p \mu(dx) < \infty.$$

Let X, Y bé Banach spaces. An operator $T \in \mathcal{L}(X, Y)$ is said to be p-Radonnifying ($p > 0$) if for each cylindrical measure Λ of type p on X, $T(\Lambda)$ is a Radon measure of order p on Y. The following theorem which will be repeatedly used in this paper establishes the relationship between p-summing and p-Radonnifying operators :

2.3.1. *Chwartz's Theorem* [10], [11]

i) If the operator T is p-Radonnifying then it is also p-summing.

ii) Conversely, if T is p-summing and $1 < p < \infty$ then T is p-Radonnifying.

2.4. Stable measures. A measure μ on X is called (symmetric) stable if for any a, $b > 0$ there exists $c > 0$ such that the ch. f. $\widehat{\mu}$ of μ satisfies the equality

$$\widehat{\mu}(ax^*)\widehat{\mu}(bx^*) = \widehat{\mu}(cx^*) \text{ for all } x^* \in X^*.$$

Below we shall list some properties of stable measures which will be used in this paper.

2.4.1. *Theorem* [12] (*The representation of the ch.f.of stable measures*)

If μ is a symmetric stable measure on X then there exist a constant $\alpha\,(0 < \alpha \leqslant 2)$ and a finite measure γ on the unit sphere S of X such that

$$\widehat{\mu}(x^*) = \exp\left\{ -\int_S |<x, x^*>|^\alpha \gamma(dx) \right\}.$$

The constant α will be called the index of μ. The measure γ will be called the spectral measure of μ.

2.4.2. ***Theorem*** [1] Every stable measure of index α $(0 < \alpha < 2)$ is of order p for each $p < \alpha$ and not of order α.

2.4.3. ***Lemma*** [4] For each $p < \alpha$ there exists a universal constant C_p such that for each stable measure μ of index α on R with the ch. f. $e^{-|bt|^\alpha}$ we have

$$\left(\int_R |t|^p \mu(dt) \right)^{1/p} = b\, C_p.$$

III — THE PROBLEM OF EXTENDING STABLE CYLINDRICAL MEASURES

Let X be a Banach space and let $T \in \mathcal{L}(X^*, L^\alpha)$ $(0 < \alpha \leqslant 2)$. Consider the functional $\lambda : X^* \to R$ defined by

$$\lambda(x^*) = \exp\{-\|Tx^*\|^\alpha\}. \tag{3-1}$$

It is essy to check that $\lambda(x^*)$ is positive definite, continuous with $\lambda(0) = 1$. Therefore $\lambda(x^*)$ is a ch. f. of some cylindrical measure on X, say $\wedge T$. Let $\mathcal{C}_\alpha(X^*, L^\alpha)$ denote the class of all operators $T \in \mathcal{L}(X^*, L^\alpha)$ such that $\wedge T$ is a Radon measure. It is easily seen that if $T \in \mathcal{C}_\alpha$, (X^*, L^α) then (3-1) is the ch. f. of a symmetric stable measure of index α. Conversely, the ch. f. of every symmetric stable measure of index α may be represented in this way, Indeed, by Theorem 2.4.1 the ch. f. of a symmetric stable measure of index α has the form

$$\widehat{\mu}(x^*) = \exp\left\{-\int_S |\langle x, x^*\rangle|^\alpha \, \Upsilon(dx)\right\}$$

$$= \exp\{-\|Tx^*\|^\alpha\},$$

where the operator $T : X^* \to L^\alpha(S, \Upsilon)$ is defined by

$$Tx^* = \langle ., x^*\rangle.$$

Our aim is to give a description of the class $\mathcal{C}_\alpha(X^*, L^\alpha)$, where the space L^α is infinitely dimensional. In what follows we shall only consider the case $1 < \alpha < 2$. We begin by proving the assertion which gives a necessary condition for T to belong to the class $\mathcal{C}_\alpha(X^*, L^\alpha)$.

3. 1. Theorem Let $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$. Then

i) T is a compact operator

ii) T is completely summing

Proof. 1) Suppose that B^* is the unit ball of X^* and (Tx_n^*) is an arbitrary sequence in TB^*. We have to show that (Tx_n^*) contains a subsequence which converges in L^*. Since B^* is $\sigma(X^*, X)$-compact (x_n) has a subsequence $(x_{n_k}^*)$ such that $(x_{n_k}^*)$ is convergent in $\sigma(X^*, X)$-topology to some x^* in B^*. So

$$\lim_{k\to\infty} \exp\{-\|Tx_{n_K}^* - Tx^*\|^\alpha\} = \lim_{k\to\infty} \exp\{-\|T(x_{n_K}^* - x^*)\|^\alpha\} =$$

$= \lim_{k\to\infty} \int_X e^{i\langle x, x_{n_K}^* - x^*\rangle} \wedge^T (dx) = 1$ by virtue of Lebesgue's dominated convergence theorem.

This implies $\|Tx_{n_K}^* - Tx^*\| \to 0$ i.e. $Tx_{n_K}^* \to Tx^*$ in L^α.

ii) It is sufficient to show that T is p-sunming for each $0 < p < \alpha$. Since x^* considered as a random variable on $(X, \mathcal{B}(X), \wedge T)$ presents itself as a stable random variable of index α with the ch. $f f(t) = \exp\{-\|Tx^*\|^\alpha |t|^\alpha\}$, by Lemma 2. 4. 3 we have

$$\|Tx^*\|^p = C_p^{-p} \int_X |<x, x^*>|^p \wedge^T (dx).$$

For each finite collection $(x_n^*) \subset X^*$ we have

$$\sum \|Tx_n^*\|^p = C_p^{-p} \int_X \sum |<x, x_n^*>|^p \wedge^T (dx) \leqslant$$

$$C_p^{-p} \int_X \|x\|^p \wedge^T (dx) \left\{ \sup_{\|x\| \leqslant 1} \sum |<x, x_n^*>|^p \right\}$$

$$= C \sup_{\|x\| \leqslant 1} \sum |<x, x_n^*>|^p$$

where we put $C = C_p^{-p} \int_X \|x\|^p \wedge^T (dx)$. $C < \infty$ because $\int_X \|x\|^p \wedge^T (dx) < \infty$. by Theorem 2.4.2.

This proves that T is p-summing.

3.2. Theorem. Suppose that $T \in \mathcal{L}(X^*, L^\alpha)$ such that the adjoint operator T^* operates from L^β $(\beta^{-1} + \alpha^{-1} = 1)$ into X. If T^* is p-summing for some $0 < p < \alpha$ then $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$.

Proof. Let $m^{(\alpha)}$ be a cylindrical measure on L^β with the ch. f. $\exp\{-\|x^*\|^\alpha\}$ $(x^* \in L^\alpha)$. $m^{(\alpha)}$ is the cylindrical measure of type p for each $p < \alpha$. Indeed, by Lemma 2.4.3 we have

$$\left(\int_R |u|^p \, m_{x^*}^{(\alpha)} (du) \right)^{1/p} = C_p \|x^*\|.$$

Thus

$$\sup_{\|x^*\| \leqslant 1} \left(\int_R |u|^p \, m_{x^*}^{(\alpha)} (du) \right)^{1/p} = C_p < \infty$$

Consequently, from the above assumption and from Schwartz's theorem it follows that $T^*(m^{(\alpha)})$ is a Radon measure on X. Evidently, $T^*(m^{(\alpha)})$ is the Radon extension of the cylindrical measure $\wedge^T$.

3.3. Theorem. i) Suppose that X can be immersed in some $L^p(1 < p \leqslant 2)$ $(X \Leftrightarrow L^p$ in short). Then, the class $\mathcal{C}_\alpha(X^*, L^\alpha)$ coincides with the class $\pi_0(X^*, L^\alpha)$ of all completely summing operators from X^* into L^α.

ii) In the case $X = L^p$ $(p > 2)$, however, the class $\mathcal{C}_\alpha(X^*, L^\alpha)$ is strictly included in the class $\pi_0(X^*, L^\alpha)$.

Proof. i) In view of Theorem 3-1 it remains for us to prove the inclusion $\pi_0(X^*, L^\alpha) \subset \mathcal{C}_\alpha(X^*, L^\alpha)$. The assumption that $X \hookrightarrow L^p (1 < p \leqslant 2)$ implies that the function $\exp\{-\|x\|_X^p\}(x \in X)$ is positive definite. Let $m^{(p)}$ denote a cylindrical measure on X^* with the ch. f. $\exp\{-\|x\|_X^p\}(x \in X)$. Suppose that $T \in \pi_0(X^*, L^\alpha)$. Since $m^{(p)}$ is of type q for each $q < p$ then by Schwartz's theorem $T(m^{(p)})$ is a Radon measure on L^α. $T(m^{(p)})$ has the ch. f. $\exp\{-\|T^*x\|^p\}(x \in L^\beta)$. According to theorem 3.1 T^* is completely summing (i.e T^* is p-summing for each $p > 0$). From this, by Theorem 3.2 it follows that $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$.

ii) We shall show that there exists an operator T such that $T \in \pi_0(X^*, L^\alpha)$ but $T \notin \mathcal{C}_\alpha(X^*, L^\alpha)$. Indeed, because $L^p(p > 2)$ cannot be immersed in any L^α, by Lindenstraus—Pelczynski's theorem [6] there exist two sequences $\{x_n\} \subset X$ and $\{y_n\} \subset X$ satisfying

$$\sum_{n=1}^{\infty} |<y_n, x^*>|^\alpha \leqslant \sum_{n=1}^{\infty} |<x_n, x^*>|^\alpha \tag{3-2}$$

for all $x^* \in X^*$ and such that $\Sigma\|x_n\|^\alpha < \infty$ $\Sigma\|y_n\|^\alpha = \infty$.

Consider the operator $T : X^* \to l^\alpha$ defined by

$$Tx^* = (<y_n, x^*>)_{n=1}^{\infty}$$

T can be considered as an operator from X^* into L^α because $l^\alpha \hookrightarrow L^\alpha$. We shall show that $T \in \pi_0(X^*, L^\alpha)$ but $T \notin \mathcal{C}_\alpha(X^*, L^\alpha)$.

a) $T \in \pi_0(X^*, L^\alpha)$: Since X is of stable-type 2, by Maurey—Pisier's Theorem it is sufficient to show that $T \in \pi_\alpha(X^*, L^\alpha)$. Consider an arbitrary sequence $(x_n^*) \subset X$ such that $\Sigma|<x, x_n^*>|^\alpha < \infty$ for each $x \in X$. Then, by the principle of uniform boundedness $\Sigma|<x, x_n^*>|^\alpha \leqslant C\|x\|^\alpha$ for some $C > 0$ and all x. Thus, we have

$$\sum_{n=1}^{\infty} \|Tx_n^*\|^\alpha = \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} |<y_k, x_n^*>|^\alpha = \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} |<y_k, x_n^*>|^\alpha \leqslant$$

$$\leqslant \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} |<x_k, x_n^*>|^\alpha \leqslant C \sum_{k=1}^{\infty} \|x_k\|^\alpha < \infty$$

That is
$$T \in \pi_\alpha(X^*, L^\alpha)$$

b) $T \notin \mathcal{C}_\alpha(X^*, L^\alpha)$: Consider the series $\sum_{n=1}^{\infty} y_n \theta_n^{(\alpha)}$ where $(\theta_n^{(\alpha)})$ are i. i. d. realvalued random variables with the ch. f. $\exp\{-|t|^\alpha\}$. Let $\varphi_n(x^*)$ be the ch.f. of the partial sum $\sum_{k=1}^{n} y_k \theta_k^{(\alpha)}$. We have

$$\lim_{n\to\infty} \varphi_n(x^*) = \lim_{n\to\infty} \exp\left\{-\sum_{k=1}^{n} |<y_k, x^*>|^\alpha\right\} = \exp\left\{-\sum_{k=1}^{\infty} |<y_k, x^*>|^\alpha\right\} =$$
$$= \exp\left\{-\|Tx^*\|^\alpha\right\}$$

Assume $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$. This means that $\exp\{-\|T x^*\|^\alpha\}$ is a ch. f. of some Radon measure. Therefore, by Ito — Nisio's theorem $\sum_{n=1}^{\infty} y_n \theta_n^{(\alpha)}$ converges a. s. Since X is of stable-cotype α (Every Banach space being of stable-cotype α with $0 < \alpha < 2$) the convergence of $\sum y_n \theta_n^{(\alpha)}$ implies $\sum_{n=1}^{\infty} \| y_n \|^\alpha < \infty$. This contradicts (3-2).

Theorem 3.3 allows us to obtain the general form of characteristic functionals of symmetric stable measures of index α in space which can be immersed in some L^p with $1 < p \leqslant 2$.

3.4. Corollary. Let X be immersed in some L^p $(1 < p \leqslant 2)$.

A functional $\lambda: X^* \to R$ is a ch. f. of a symmetric stable measure of index α $(1 < \alpha \leqslant 2)$ if and only if it may be represented in the form

$$\lambda(x^*) = \exp\left\{-\|Tx^*\|^\alpha_{L^\alpha}\right\},$$

where $T: X^* \to L^\alpha$ is a completely summing operator.

It is known that [3], in the case $\alpha = 2$ the equality $\mathcal{C}_2(X^*, L^2) \equiv \pi_2(X^*, L^2)$ holds if and only if X is of type 2. The following theorem characterizes those spaces for which the equality $\mathcal{C}_\alpha(X^*, L^\alpha) \equiv \pi_\alpha(X^*, L^\alpha)$ holds.

3.5. Theorem. Let X be a Banach space. Then the following assertions are equivalent:

i) X can be immersed in some L^α and is of stable-type α.

ii) $$\mathcal{C}_\alpha(X^*, L^\alpha) \equiv \pi_\alpha(X^*, L^\alpha).$$

Proof. i) → ii) Since $X \hookrightarrow L^\alpha$, by theorem 3.3 we have $\mathcal{C}_\alpha(X^*, L^\alpha) \equiv \pi_0(X^*, L^\alpha)$. On the other hand, since X is of stable-type α it follows from Maurey-Pisier's theorem that $\pi_0(X^*, L^\alpha) \equiv \pi_\alpha(X^*, L^\alpha)$ Therefore $\mathcal{C}_\alpha(X^*, L^\alpha) \equiv \pi_\alpha(X^*, L^\alpha)$.

ii) → i) As we have shown in the proof of the part ii) of theorem 3.3 if X cannot be immersed in any L^α then there exists an operator T such that $T \in \pi_\alpha(X^*, L^\alpha)$ but $T \notin \mathcal{C}_\alpha(X^*, L^\alpha)$. Consequently, X can be immersed in some L^α.

Next, we show that X is of stable-type α. Let $(x_n) \subset X$, $\Sigma \| x_n \|^\alpha < \infty$. We have to prove that the series $\sum_{n=1}^{\infty} x_n \theta_n^{(\alpha)}$ converges a.s. Consider the operator $T: X^* \to l^\alpha$ defined by $Tx^* = \{\langle x_n, x^* \rangle\}_{n=1}^{\infty}$. T can be considered as an operaor from X^* into L^α because $l^\alpha \hookrightarrow L^\alpha$. From Ito—Nisio's theorem it easily follows that the series $\Sigma x_n \theta_n^{(\alpha)}$ will converge a.s. if we show that $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$ i.e $T \in \pi_\alpha(X^*, L^\alpha)$ by our assumption. Consider an arbitrary sequence $(x_n^\alpha) \subset X^*$ such that $\Sigma \mid \langle x, x_n^* \rangle \mid^\alpha < \infty$ for each $x \in X$. By the uniform boundedness principle $\Sigma \mid \langle x, x_n^* \rangle \mid^\alpha \leqslant C \| x \|^\alpha$ for some $C > 0$ and all $x \in X$. Thus we have $\sum_k \| Tx_k^* \| = \sum_k \sum_n \mid \langle x_n, x_k^* \rangle \mid^\alpha = \sum_n \sum_k \mid \langle x_n, x_k^* \rangle \mid^\alpha \leqslant C \sum_n \| x_n \|^\alpha < \infty$. That is $T \in \pi_\alpha(X^*, L^\alpha)$.

Remark. Taking into account theorems 3.4 and 3.5 the following problem arises quite naturally: Characterize those Banach spaces in which the class $\mathcal{C}_\alpha(X^*, L^\alpha)$ coincides with the class $\pi_\alpha(X^*, L^\alpha)$ of all completely summing operators from X^* into L^α.

Let $\Gamma_\alpha(X)$ denote the class of all spectral measures corresponding to symmetric stable measures on X with index α. It is known that if X is of stable-type α then $\Gamma_\alpha(X)$ consists of all finite measures on the unit sphere S (see [3]). In particular, we get the description of $\Gamma_\alpha(X)$ for the case $X = l^p (p > \alpha)$. Here, as an application of above—obtained results, we shall prove the following theorem which gives a description of the class $\Gamma_\alpha(X)$ for the case $X = l^p (1 < p < \alpha)$ (Note that for arbitrary X the problem of determining $\Gamma_\alpha(X)$ still remains open).

3.7. Theorem. Let $X = l^p (1 < p < \alpha)$. Then the class $\Gamma_\alpha(X)$ consists of finite measures γ on S satisfying the condition

$$\sum_{n=1}^{\infty} \left[\int_S \mid \langle x, e_n \rangle \mid^\alpha \gamma(dx) \right]^{p/\alpha} < \infty,$$

where (e_n) is the sequence of unit vectors in $X^* = l^q$.

Proof. We let $L^\alpha = L^\alpha(S, \gamma)$. Consider the operator $T: X^* \to L^\alpha$ defined by $Tx^* = \langle ., x^* \rangle$. Then the fact that $\gamma \in \Gamma_\alpha(X)$ is equivalent to the fact that $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$. Therefore the theorem will be proved as soon as we show that $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$ if and only if $\Sigma \| Te_n \|^p < \infty$. Suppose that $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$. By theorem 3.1 $T \in \pi_p(X^*, L^\alpha)$. From this it follows that $\Sigma \| Te_n \|^p < \infty$ since (e_n) is a weakly p-absolutely summable sequence in $X^* = l^q$. Conversely, suppose that $\Sigma \| Te_n \|^p < \infty$. Consider the series $\sum_n T(e_n)\theta_n^{(p)}$ where $(\theta_n^{(p)})$ are i.i.d. real-valued random variables with the ch. f. $\exp\{- \mid t \mid^p\}$. Because L^α is of stable-type p $(p < \alpha)$, $\Sigma \| Te_n \|^p < \infty$ implies that the series $\Sigma T(e_n)\theta_n^{(p)}$ converges a.s in L^α. This fact implies that $\exp\{- \| T^* x^* \|_X^p\}$ is the ch. f. of a Radon measure on L^α. According to theorem 3.1, $T^* \in \pi_0(L^\beta, X)$ and, consequently, $T \in \mathcal{C}_\alpha(X^*, L^\alpha)$ by theorem 3.2.

Received May 15th 1978

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