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Commutative subalgebras and representations of the envelopping algebra of a solvable lie algebra(*)

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Let $\mathcal{G}$ be a solvable ρ-dimensional complex Lie algebra, $\mathcal{U}(\mathcal{G})$ its envelopping algebra and $\mathcal{K}(\mathcal{G})$ its envelopping field. In the present work, we construct some prime ideals $\mathcal{J}$ of $\mathcal{U}(\mathcal{G})$ and commutative subalgebras of $\mathcal{U}(\mathcal{G})/\mathcal{J}$ (such an algebraic tool was used in [1, 2] for the harmonic analysis on the Inhomogeneous Lorentz group).

Two previous papers [3,4] are helpful for the present purpose. In [3] are constructed commutative subfields K of $\mathcal{K}(\mathcal{G})$ and in [4], the subalgebras $a = K \cap \mathcal{U}(\mathcal{G})$ and their characters are used to produce irreducible representations of $\mathcal{U}(\mathcal{G})$. The results and the proofs given there give a good account of what will be done here.

I. SPECIAL SUBALGEBRAS.

(1) Let $\mathcal{B}$ be a ring. A *non commutative polynomial* $P(X_1, ..., X_\sigma)$ over $\mathcal{B}$ should be a polynomial in the sense of [3] II. 1 with coefficients in $\mathcal{B}$ (not necessary commutative): the indeterminates $X_1, ..., X_\sigma$ are not allowed to commute one to another, nor to the coefficients. One says that $P(X_1, ..., X_\sigma)$ is right-ordered if

$$P(X_1, ..., X_\sigma) = \sum_\alpha X_1^{\alpha_1} \dots X_\sigma^{\alpha_\sigma} a_\alpha.$$

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Let $\mathcal{J}$ be a given ideal in $\mathcal{B}$. To each $P(X_1, \ldots, X_\sigma)$, one associates its *dominant commutative modulo $\mathcal{J}$ part* $P^*(x_1, \ldots, x_\sigma)$ by keeping in $P(X_1, \ldots, X_\sigma)$ only the monomials of highest degree, replacing there each coefficient $\in \mathcal{B}$ by its image in $\mathcal{B}/\mathcal{J}$, each X_i by x_i and by allowing the x_i's to commute to everything.

(2) Let $\mathcal{B} \subset \mathcal{A}$ be rings and $\mathcal{J}$ an (two-sided) ideal in $\mathcal{B}$. One says that $a_1, \ldots, a_\sigma \in \mathcal{A}$ are *algebraically* (resp. *right-algebraically*) *independent over $\mathcal{B}$ modulo $\mathcal{J}$* if $P(a_1, \ldots, a_\sigma) \notin \mathcal{J}$ for every non commutative polynomial $P(X_1, \ldots, X_\sigma)$ over $\mathcal{B}$ such that $P^*(x_1, \ldots, x_\sigma) \neq 0$ (resp. which is right-ordered and possesses a coefficient not in $\mathcal{J}$).

These definitions generalize [3] II.3 and the following generalizes [3] III.1:

(3) *DEFINITION.*

Let $\mathcal{J}$ be a two-sided ideal in $\mathcal{U}(\mathcal{G})$ and $\mathcal{B} \subset \mathcal{U}(\mathcal{G})$ a subalgebra containing $\mathcal{J}$. Consider the following assertions:

$\mathcal{A}_1$. *There exist $B_1, \ldots, B_N \in \mathcal{B}$, $E \in \mathcal{B} \setminus \mathcal{J}$, $T_1, \ldots, T_\delta \in \mathcal{G}$ such that*

(i) *$\mathcal{B}/\mathcal{J}$ is comutative, generated over $\mathbf{C}$ by $B_1 + \mathcal{J}, \ldots, B_N + \mathcal{J}$ and contains no zero divisors of $\mathcal{U}(\mathcal{G})/\mathcal{J}$ (except 0).*

(ii) $[\mathcal{G}, \mathcal{B}] \subset \mathcal{B}$.

(iii) *$T_1, \ldots, T_\delta$ are algebraically independent over $\mathcal{B}$ modulo $\mathcal{J}$.*

(iv) $\mathcal{G}E = \sum_{i=0}^{\delta} T_i \mathcal{B}$ *(with $T_0 = 1 \in \mathcal{U}(\mathcal{G})$).*

$\mathcal{A}_2$. *In the notations of $\mathcal{A}_1$, one has for $j = 1, \ldots, N$,*

$$B_j = \sum_{i=1}^{\rho} T_i \, b_j^i,$$

where $(T_1, \ldots, T_\rho)$ is a basis of $\mathcal{G}$ which completes $(T_1, \ldots, T_\delta)$ and each b_j^i is a right ordered polynomial in $B_1, \ldots, B_{j-1}$ over $\mathbf{C}$.

$\mathcal{A}_3$. *In the notations of $\mathcal{A}_1$, the rank of the matrix*

$$\mathcal{M}_\delta = \left([T_i, B_j] + \mathcal{J}\right) \qquad i = 1, \ldots, \delta; \quad j = 1, \ldots, N$$

is equal to δ.

If $\mathcal{A}_1$ is (resp. and $\mathcal{A}_2$ are) true, $\mathcal{B}$ is said to be of S-type (resp. S'-type) on $\mathcal{J}$ and if furthermore $\mathcal{A}_3$ is true, $\mathcal{B}$ is said to be quasimaximal.

Remark : When $\mathcal{B}$ is of S'-type, in fact the elements

$$B_j^{\otimes} = \sum_{i=1}^{\rho} T_i \otimes b_j^i \in \mathcal{G} \otimes_{\mathbb{C}} \mathcal{B} \quad \text{for } j = 1, \ldots, N$$

are given by definition.

In the following, we shall construct these algebras.

The definition $(\mathcal{A}_1)$ follows [3] III. 1 very closely. In $(\mathcal{A}_2)$ we collected the special features of the explicit construction we give and these properties will prove to be useful in the study of representations. The definition $(\mathcal{A}_3)$ can be understood from [4] II. 2.

The algebraic structure underlying in $(\mathcal{A}_1)$ is interesting. To make it clear, let us introduce the following

(4) *DEFINITION.* (Two-sided Lie Algebras)

Let k be a commutative field and Γ a commutative k-algebra with unit. A two-sided Lie algebra $(\mathfrak{h}, \delta, T_0)$ over (k, Γ) is defined to have the following properties :

(i) *$\mathfrak{h}$ is a Γ-bimodule of finite type.*

(ii) *$\mathfrak{h}$ is a Lie-algebra over k.*

(iii) $\delta : \mathfrak{h} \to \mathrm{Der}_k\, \Gamma$(*) *is two-sided Γ-linear and is a Lie-algebra homomorphism such that*

$$[\lambda X, \mu Y] = \lambda\mu [X, Y] + \lambda\big(\delta(X) \cdot \mu\big) Y - \big(\delta(Y) \cdot \lambda\big)\mu X,$$

$$[X\lambda, \mu Y] = [X, Y]\lambda\mu + Y\lambda\big(\delta(X) \cdot \mu\big) - X\big(\delta(Y) \cdot \lambda\big)\mu,$$

$$T_0 \in \mathfrak{h} \setminus \{0\},\ [\mathfrak{h}, T_0] = 0,\ \delta(T_0) = 0,$$

$$[X\lambda - \lambda X] = T_0\, \delta(X) \cdot \lambda$$

for all $\lambda, \mu \in \Gamma$ and $X, Y \in \mathfrak{h}$.

As for the classical Lie-algebras, let $\mathcal{C} = \Gamma \oplus \mathfrak{h} \oplus \mathfrak{h} \otimes_\Gamma \mathfrak{h} \oplus \ldots$ be the tensor-algebra of the Γ-bimodule $\mathfrak{h}$ and $\mathcal{J}$ the ideal in $\mathcal{C}$ generated by $T_0 - 1$ and the elements $X \otimes Y - Y \otimes X - [X, Y]$ (where $X, Y \in \mathfrak{h}$). Define $\mathcal{E}(\mathfrak{h}) = \mathcal{C}/\mathcal{J}$. Then one has a generalization of the envelopping algebra and the envelopping field. As in $\mathcal{E}(\mathfrak{h})$ $T_0 = 1$, we call $\mathcal{E}(\mathfrak{h})$ the reduced envelopping algebra of $\mathfrak{h}$ and we denote by $\mathcal{F}(\mathfrak{h})$ its quotient field. In $\mathcal{E}(\mathfrak{h})$, one has a filtration and the Poincaré-Birkhoff-Witt theorem generalizes to this case.

(*) $\mathrm{Der}_k\, \Gamma = \{k\text{-derivations of } \Gamma\}$

To have an example of such a situation, one may take for $\mathfrak{b}$ a closed family of vector-fields over $\mathbb{C}^N$ with polynomial coefficients, $k = \mathbb{C}$, $\Gamma = \mathbb{C}[z_1, ..., z_N]$ with the natural action δ of the vector fields on Γ and T_0 as the identity operator : $\mathcal{E}(\mathfrak{b})$ can be viewed as the algebra of differential operators generated by $\mathfrak{b}$ and the filtration is equal to the degree of the differential operator.

The verification of the following proposition is left to the reader

(5) *PROPOSITION* (notations of I.4 $\mathcal{A}_1$).

Let $\mathcal{B}$ *be of S-type on* $\mathcal{I}$ *and set* $\Gamma = \mathcal{B}/\mathcal{I}$, $\Delta =$ Fract Γ. *Then* :

(i) Γ *is a noetherian integral ring.*

(ii) $\mathcal{U}(\mathcal{G})/\mathcal{I}$ *is integral.*

(iii) *On* $\mathcal{G}_\Gamma^- = (\mathcal{G}\mathcal{B} + \mathcal{B})/\mathcal{I}$ *there is a natural structure of two-sided Lie-algebra over* $(\mathbb{C}, \Gamma)$ *(with the Lie bracket in* $\mathcal{U}(\mathcal{G})/\mathcal{I}$*).*

(iv) $\mathcal{G}_\Delta^- = \mathcal{G}_\Gamma^- \otimes_\Gamma \Delta$ *is a two-sided Lie algebra over* $(\mathbb{C}, \Delta)$ *whose a basis is* $(1 + \mathcal{I}, T_1 + \mathcal{I}, ..., T_\delta + \mathcal{I})$ *and* $\mathcal{G}_\Gamma^- \subsetneq \mathcal{G}_\Delta^-$.

(v) $\mathcal{U}(\mathcal{G})/\mathcal{I} \simeq \mathcal{E}(\mathcal{G}_\Gamma^-)$ *(as* $\mathbb{C}$*-algebras and* Γ*-bimodules).*

$\mathcal{U}(\mathcal{G})/\mathcal{I} \subsetneq \mathcal{E}(\mathcal{G}_\Delta^-)$ *(and* Fract $\mathcal{U}(\mathcal{G})/\mathcal{I} = \mathcal{F}(\mathcal{G}_\Delta^-)$*).*

(vi) *Let us denote by the subscript* **E** *the localization at* **E***; then*

$$(\mathcal{U}(\mathcal{G})/\mathcal{I}) \subsetneq (\mathcal{U}(\mathcal{G})/\mathcal{I})_E \simeq \mathcal{E}(\mathcal{G}_{\Gamma_E}^-).$$

Here, $\mathcal{G}_{\Gamma_E}^- = \mathcal{G}_\Gamma^- \otimes_\Gamma \Gamma_E$ *is a two-sided Lie-algebra over* $(\mathbb{C}, \Gamma_E)$ *with* $(1 + \mathcal{I}, T_1 + \mathcal{I}, ..., T_\delta + \mathcal{I})$ *as a basis.*

This proposition gives the isomorphism between $(\mathcal{U}(\mathcal{G})/\mathcal{I})_E$ and $\mathcal{E}(\mathcal{G}_{\Gamma_E}^-)$. This last algebra, when $\delta T_1, ..., \delta T_\delta \in \mathrm{Der}_{\mathbb{C}}\Gamma_E$ are Γ_E-linearly independent, can be identified with the differential operator over Γ_E generated by the «first degree differential operators» $T_1, ..., T_\delta$.

II. CONSTRUCTION OF THE SPECIAL ALGEBRAS.

(1) Let $\mathcal{B}$ be a subalgebra of $\mathcal{U}(\mathcal{G})$ such that $[\mathcal{G}, \mathcal{B}] \subset \mathcal{B}$. Denote by $\hat{\mathcal{B}}$ the set of characters of $\mathcal{B}$. Define Φ (resp. $f^{\otimes}$ for any $f \in \hat{\mathcal{B}}$) to be the $\mathcal{B}$-linear (resp. $\mathbb{C}$-linear) map : $\mathcal{G} \otimes_{\mathbb{C}} \mathcal{B} \to \mathcal{U}(\mathcal{G})$ such that $\Phi(T \otimes b) = Tb$ (resp. $f^{\otimes}(T \times b) = \langle f, b \rangle T$) for all $T \in \mathcal{G}$ and $b \in \mathcal{B}$. Set $\mathcal{B}^{\otimes} = \Phi^{-1}(\mathcal{B})$

and $b_f = f^{\otimes}(\mathcal{B}^{\otimes}) \subset \mathcal{G}$. If $f \mid \Phi(\mathcal{B}^{\otimes} \cap \ker f^{\otimes}) = 0$ then f is said to be compatible. In this case, $f \circ \Phi \mid \mathcal{B}^{\otimes}$ is zero on $\ker(f^{\otimes} \mid \mathcal{B}^{\otimes})$ and defines a linear form χ_f on $b_f \simeq \mathcal{B}^{\otimes}/\ker(f^{\otimes} \mid \mathcal{B}^{\otimes})$.

$$\begin{array}{ccccc} \mathcal{B}^{\otimes} & \xrightarrow{\Phi} & \mathcal{B} & \xrightarrow{f} & \mathbf{C} \\ {\scriptstyle f^{\otimes}}\downarrow & & & & \\ \mathcal{G} & & & & \end{array}$$

Let k_f be the stabilizator of f in $\mathcal{G}$, i.e. $k_f = \{X \in \mathcal{G},\ f \mid [X, \mathcal{B}] = 0\}$. The character f of $\mathcal{B}$ defines an 1-dimensional $\mathcal{B}$-module V_f where one takes a non zero vector $\hat{f}$. The induced $\mathcal{G}$-module $W_f = \mathcal{U}(\mathcal{G}) \otimes_{\mathcal{B}} V_f$ is then generated by $1 \otimes \hat{f}$. The character f is compatible whenever $W_f \neq 0$ and for any $H \in b_f$ one has then

$$H(1 \otimes \hat{f}) = \langle \chi_f, H \rangle\ 1 \otimes \hat{f},$$

that is, $1 \otimes \hat{f}$ is an eigenvector for the action of b_f and χ_f is the eigenvalue. Furthermore, if f is compatible, then b_f is an ideal of k_f such that $\chi_f \mid [k_f, b_f] = 0$ and χ_f is a character of b_f (see [4] V.2 and V.3.1). Set $\mathcal{N}_f = \operatorname{Ann} W_f$.

(2) *DEFINITION.*

Let $\mathcal{B}$ be a subalgebra of S-type (resp. S'-type) on $\mathcal{J}$ in the notations of I.4. A character f of $\mathcal{B}/\mathcal{J}$ is said to be of S-type (resp. S'-type) if for any basis $(Y_1, \ldots, Y_\sigma)$ of any vector space supplementary to b_f

$$\left(\textit{resp. to}\quad b_f^0 = \sum_{j=1}^{N} \mathbf{C} \sum_{i=1}^{\rho} \langle f, b_j^i \rangle\, T_i\right)$$

$Y_1, \ldots, Y_\sigma$ are right algebraically independent over $\mathcal{B}$ modulo ker f.

When f is of S'-type, f is of S-type. Suppose that f is of S-type, then W_f admits $\{Y_1^{\alpha_1} \ldots Y_\sigma^{\alpha_\sigma} \otimes \hat{f},\ \alpha = (\alpha_1, \ldots, \alpha_\sigma) \in \mathbf{N}^\sigma\}$ as a $\mathbf{C}$-basis if $(Y_1, \ldots, Y_\sigma)$ is a basis of any vector space supplementary to b_f. It is clear that W_f is then isomorphic to the $\mathcal{U}(\mathcal{G})$-module induced by $\chi_f \in \widehat{\mathcal{U}(b_f)}$.

The module W_f is simple if and only if $b_f = k_f$. As one is interested only in simple module, special care must be taken for the characters of $\mathcal{B}/\mathcal{J}$ which are compatible and such that $b_f \neq k_f$ (remember that $b_f \subset k_f$).

Let $\mathcal{B}$ be of S-type on $\mathcal{J}$; when f runs in $\widehat{\mathcal{B}/\mathcal{J}}$, $\rho - \delta$ is the supremun of the dimension of b_f (δ is given in $\mathcal{A}_1$). When $\dim b_f = \rho - \delta$, f is of S-type and for any basis $(Y_1, \ldots, Y_\delta)$ of any vector space supplementary to b_f, $Y_1, \ldots, Y_\sigma$ are right algebraïcally independent over $\mathcal{B}$ modulo $\mathcal{J}$.

It may happen that $\dim b_f < \rho - \delta$ or $\dim k_f > \rho - \delta$. To handle these characters we

(3) *Blow up the ideal* $\mathcal{J}$.

Let $\mathcal{G}$ be the adjoint group of $\mathcal{g}$. Then $\mathcal{G}$ acts in $\mathcal{B}$, in $\mathcal{J}$, in $\mathcal{B}/\mathcal{J}$ and in $(\mathcal{B}/\mathcal{J})^\wedge$. Furthermore, an inner automorphism preserves the numbers dim b_f, dim k_f and the quality of being compatible or of S-type.

Set $\mathcal{X}(\mathcal{B}, \mathcal{J}) = \{f \in (\mathcal{B}/\mathcal{J})^\wedge;\ b_f = k_f\}$; clearly $\mathcal{X}(\mathcal{B}, \mathcal{J})$ is an open $\mathcal{G}$-stable subset of $(\mathcal{B}/\mathcal{J})^\wedge$.

If Ω is a constructible $\mathcal{G}$-stable subset of $(\mathcal{B}/\mathcal{J})^\wedge$, $\Omega^s = \Omega \setminus \mathcal{X}(\mathcal{B}, \mathcal{J})$ is also $\mathcal{G}$-stable and we may decompose it into a set $\mathcal{V}$ of $\mathcal{G}$-stable « irreducible components » by the following rules:

(i) Include all irreducible components of $(\Omega^s)^-$ (= closure of Ω^s).

(ii) If $V \in \mathcal{V}$, include all the irreducible components of $\big((V \setminus \Omega^s)^- \cap \Omega^s\big)^-$.

(iii) If V and $V' \in \mathcal{V}$, include all the irreducible components of

$$(V \cap V' \cap \Omega^s)^-.$$

(iv) If $V \in \mathcal{V}$, set $V_2 = \{f \in V;\ \dim b_f < \sup\{\dim b_g;\ g \in V\}\}$ and include all the irreducible components of $(V_2 \cap \Omega^s)^-$.

Obviously $\mathcal{V}$ is a finite set of subvarieties of $(\mathcal{B}/\mathcal{J})^\wedge \setminus \mathcal{X}(\mathcal{B}, \mathcal{J})$ and for each $V \in \mathcal{V}$, Ω^s is dense in V. The null ideal $\mathcal{Z}$ of V in $\mathcal{B}$ contains strictly $\mathcal{J}$. As each $V \in \mathcal{V}$ is $\mathcal{G}$-stable, $[\mathcal{g}, \mathcal{Z}] \subset \mathcal{Z}$.

In the application, Ω will be a set of characters of $\mathcal{B}/\mathcal{J}$ of S-type. If $f \in \Omega$, there exists a minimal $V \in \mathcal{V}$ which contains f. This character can then be viewed as a character of a bigger algebra $\mathcal{C}$ which is defined below with $\mathcal{Z} =$ = null ideal of V.

(4) *PROPOSITION* (to change the ideal).

Let $\mathcal{J}$ be a two-sided ideal in $\mathcal{U}(\mathcal{g})$, $\mathcal{B}$ a subalgebra of $\mathcal{U}(\mathcal{g})$ of S-type (resp. S'-type) on $\mathcal{J}$ and $\mathcal{Z}$ a prime ideal in $\mathcal{B}$ such that

$\mathcal{J} \subset \mathcal{Z}$, $[\mathcal{g}, \mathcal{Z}] \subset \mathcal{Z}$ and $(\mathcal{B}/\mathcal{Z})^\wedge$ contains a dense open set V_0 of S-type characters of $\mathcal{B}/\mathcal{J}$.

Set $\mathcal{K} = \{a \in \mathcal{U}(\mathcal{g}),\ a(\mathcal{B} \setminus \mathcal{J}) \cap \mathcal{U}(\mathcal{g})\mathcal{Z} \neq \varnothing\}$ and $\mathcal{C} = \mathcal{B} + \mathcal{K}$. Then

(i) *$\mathcal{K}$ is a prime ideal in $\mathcal{U}(\mathcal{g})$ such that $\mathcal{K} \cap \mathcal{B} = \mathcal{Z}$.*

(ii) *If $\mathcal{L}$ is a prime ideal in $\mathcal{U}(\mathcal{g})$ such that $\mathcal{L} \cap \mathcal{B} = \mathcal{Z}$, then $\mathcal{K} \subset \mathcal{L}$.*

(iii) *$\mathcal{C}$ is a subalgebra of S-type (resp. S'-type) on $\mathcal{K}$:*

In the notations of I.4, there exists an integer $\sigma \geqslant \delta$, a basis $(Y_1, \ldots, Y_\rho)$

of $\mathcal{G}$ *and* $E_0 \in \mathcal{B} \setminus \mathcal{Z}$ *such that the assertion* $(\mathcal{A}_1)$ is *(resp. and* $\mathcal{A}_2$ *are) true if one replaces each* T *by* Y, *each* $\mathfrak{s}$ *by* σ, $\mathcal{B}$ *by* $\mathcal{C}$, $\mathcal{J}$ *by* $\mathcal{K}$, E *by* E_0 *(resp. and the* b_j^i*'s by the* a_j^i*'s defined by* $\sum_{i=1}^{\rho} T_i \otimes b_j^i = \sum_{i=1}^{\rho} Y_i \otimes a_j^i$).

(iv) *The canonical map from* $\mathcal{B}/\mathcal{Z}$ *into* $\mathcal{C}/\mathcal{K}$ *is an isomorphism and defines an isomorphism from* $(\mathcal{B}/\mathcal{Z})^\wedge$ *into* $(\mathcal{C}/\mathcal{K})^\wedge$ *denoted by* $\bar{\omega}$.

(v) *One has*

$$\rho - \sigma = \operatorname{Sup}\left\{\dim \mathfrak{b}_f;\ f \in (\mathcal{B}/\mathcal{Z})^\wedge\right\} = \operatorname{Sup}\left\{\dim \mathfrak{b}_g;\ g \in (\mathcal{C}/\mathcal{K})^\wedge\right\}.$$

If $f \in (\mathcal{B}/\mathcal{Z})^\wedge$ *and* $\dim \mathfrak{b}_f = \rho - \sigma$, *then* f *is an S-type character of* $\mathcal{B}/\mathcal{J}$, $\bar{\omega}f$ *is an S-type character of* $\mathcal{C}/\mathcal{K}$, $\mathfrak{b}_f = \mathfrak{b}_{\bar{\omega}f}$ *and* $\chi_f = \chi_{\bar{\omega}f}$.

(vi) *Set* $\Omega' = \bar{\omega}\,\{f \in (\mathcal{B}/\mathcal{Z})^\wedge;\ \dim \mathfrak{b}_f = \rho - \sigma\}$. *Then* Ω' *is a* $\mathcal{G}$*-stable open dense subset of* $(\mathcal{C}/\mathcal{K})^\wedge$ *built up by S-type characters. If* $\mathcal{B}$ *is of S'-type on* $\mathcal{J}$, $f \in (\mathcal{B}/\mathcal{J})^\wedge$ *and* $\varphi \in \mathcal{G}^*$ *fulfil* $\dim \mathfrak{b}_f = \rho - \sigma$ *and* $\varphi|\mathfrak{b}_f = \chi_f$, *then there exists a unique* $f' \in \Omega'$ *such that* $\varphi|\mathfrak{b}_{f'} = \chi_{f'}$. *One has furthermore* $f' = \bar{\omega}f$, $\mathfrak{b}_{f'} = \mathfrak{b}_f$ *and* $\chi_{f'} = \chi_f$.

(vii) *Let* $\mathcal{L}$ *be a prime ideal in* $\mathcal{U}(\mathcal{G})$ *such that* $\mathcal{Z} \subset \mathcal{L} \cap \mathcal{B}$, V_0 *is dense in* $(\mathcal{B}/_{\mathcal{L}\cap\mathcal{B}})^\wedge$ *and* $\operatorname{Sup}\left\{\dim \mathfrak{b}_f; f \in (\mathcal{B}/\mathcal{Z})^\wedge\right\} = \operatorname{Sup}\left\{\dim \mathfrak{b}_g,\ g \in (\mathcal{B}/_{\mathcal{L}\cap\mathcal{B}})^\wedge\right\}$ *then* $\mathcal{K} \subset \mathcal{L}$.

(viii) *Suppose furthermore that* $\mathcal{B}$ *is quasimaximal and* $V_0 = \mathcal{X}(\mathcal{B}, \mathcal{J}) \cap (\mathcal{B}/\mathcal{Z})^\wedge$. *Then* $\mathcal{C}$ *is also quasimaximal and with the hypothesis of* (ii), *one has* $\mathcal{K} = \mathcal{L}$.

One chooses an $f \in V_0$ with maximal $\mathfrak{b}_f$'s dimension. Let $(Y_1, \ldots, Y_\rho)$ be a basis of $\mathcal{G}$ such that $(Y_1, \ldots, Y_\sigma)$ is a basis for a vector space supplementary to $\mathfrak{b}_f$ in $\mathcal{G}$ and take $s = \rho - \sigma$ elements

$$A_i^{\otimes} = \sum_{i=1}^{\rho} Y_i \otimes a_j^i \in \mathcal{B}^{\otimes} \quad (j = 1, \ldots, s)$$

whose images

$$f^{\otimes}(A_i^{\otimes}) = \sum_{i=1}^{\rho} \langle f, a_j^i \rangle\ Y_i \quad (j = 1, \ldots, s)$$

span $\mathfrak{b}_f$.

Using the Cramer's rules and the equations

$$\sum_{i=1}^{\rho} Y_i a_j^i = A_i \in \mathcal{B} \quad (j = 1, \dots, s)$$

one can expand $Y_{\sigma+1}E_0, \dots, Y_\rho E_0$ into $\mathcal{B}$-linear combinations of $1, Y_1, \dots, Y_\sigma$ if $E_0 = \det(a_j^i)_{i=\sigma+1,\dots,\rho;\, j=1,\dots,s}$. One has $\langle f, E_0 \rangle \neq 0$ and therefore $E_0 \in \mathcal{B} \setminus \mathcal{Z}$.

The proof of the proposition is now purely computational and relies on combinatorics generalizing [3] III.2.

Whenever the new algebra is not quasimaximal, i.e. if for all $f \in V_0$ $b_f \neq k_f$, one can use the following

(5) *PROPOSITION* (to enlarge the algebra).

Let $\mathcal{B}$ be a subalgebra of S'-type on $\mathcal{J}$. In the notations of I.4 set $\mathcal{B}_0 = \mathcal{B}$ and $N_0 = N$. Then there exists an integer $p \geq 0$ and subalgebras $\mathcal{B}_0 \subset \mathcal{B}_1 \subset \dots \subset \mathcal{B}_p$ in $\mathcal{U}(\mathcal{G})$ with the following properties. Let $k \in \{1, 2, \dots, p\}$, then:

(i) *$\mathcal{B}_k$ is of S'-type on $\mathcal{J}$ and is quasimaximal if $k = p$:*

There exist integers $N_p > \dots > N_1 > N$, b_j^i's belonging to $\mathcal{U}(\mathcal{G})$ and $E_k \in \mathcal{B}_k \setminus \mathcal{J}$ such that I.4 is true when one replaces $\mathcal{B}$ by $\mathcal{B}_k$, N by N_k, E by E_k and δ by δ-k.

(ii) *Let $f \in (\mathcal{B}_k/\mathcal{J})^\wedge$. Suppose that $f \mid \mathcal{B}_{k-1}/\mathcal{J}$ is of S-type and*

$\langle f, B_j \rangle = \sum_{i=1}^{\rho} t_i \langle f, b_j^i \rangle$ $(j = 1, \dots, N_k)$ for some t_i's in $\mathbb{C}$. Then f is an S'-type character of $\mathcal{B}_k/\mathcal{J}$.

(iii) *Let $g \in (\mathcal{B}_{k-1}/\mathcal{J})^\wedge$ and $\varphi \in \mathcal{G}^*$. Suppose that g is of S-type and $\varphi \mid b_g = \chi_g$. Then there exists a unique compatible character f of $\mathcal{B}_k/\mathcal{J}$ such that $\varphi \mid b_f = \chi_f$. Furthermore f is of S'-type and $f \mid \mathcal{B}_{k-1} = g$.*

(iv) *Let Ω_{k-1} be a constructible set of S-type characters of $\mathcal{B}_{k-1}/\mathcal{J}$ and $\Omega_k = \{f \in (\mathcal{B}_k/\mathcal{J})^\wedge ; f$ is compatible and $f \mid \mathcal{B}_{k-1}/\mathcal{J} \in \Omega_k\}$. Then Ω_k is a constructible set of S'-type characters.*

Let $\mathcal{L}$ be a prime ideal in $\mathcal{B}$. Suppose $\mathcal{J} \subset \mathcal{L}$ and Ω_{k-1} dense in $(\mathcal{B}_{k-1}/\mathcal{B}_{k-1} \cap \mathcal{L})^\wedge$. Then Ω_k is dense in $(\mathcal{B}_k/\mathcal{B}_k \cap \mathcal{L})^\wedge$.

(v) *Let Ω' be a constructible set of S-type characters of $\mathcal{B}/\mathcal{J}$, $\mathcal{L}$ be a prime ideal in $\mathcal{U}(\mathcal{G})$. Set $\mathcal{B}' = \mathcal{B}_p$ and $\Omega'' = \{f \in (\mathcal{B}'/\mathcal{J})^\wedge ; f$ is compatible and $f \mid \mathcal{B}/\mathcal{J} \in \Omega'\}$. Then ($\mathcal{B}'$ is quasimaximal of S'-type on $\mathcal{J}$ and) Ω'' is a constructible* *set of S'-type characters of $\mathcal{B}'/\mathcal{J}$ which is dense in $(\mathcal{B}'/\mathcal{B}' \cap \mathcal{L})\widehat{}$ whenever Ω' is dense in $(\mathcal{B}/\mathcal{B} \cap \mathcal{L})\widehat{}$.*

Let g be an S-type character of $\mathcal{B}/\mathcal{J}$ and $\varphi \in \mathcal{G}^*$ be such that $\varphi | b_g = \chi_g$. Then there exists a unique compatible character f of $\mathcal{B}'/\mathcal{J}$ such that $\varphi | b_f = \chi_f$. Furthermore, f is of S-type and $f | \mathcal{B} = g$.

This proposition is proved in two steps : One constructs first the growing sequence $\mathcal{B} \subset \mathcal{B}_1 \subset ... \subset \mathcal{B}_p$ of subalgebras of $\mathcal{U}(\mathcal{G})$ and then proves the required properties. We shall omit the second part as it is very long and computational.

The main scheme of the construction is given by [3] III.4.

With some care, this construction generalizes to the case of the envelopping field of $\mathcal{L} = (\mathcal{G}\mathcal{B} + \mathcal{B})/\mathcal{J}$ and one can exhibit an increasing sequence of subfields K_k with analogous properties as in [3] III.4 and III.9.

We know that the reduced envelopping algebra $\mathcal{E}(\mathcal{L})$ is isomorphic to $\mathcal{U}(\mathcal{G})/\mathcal{J}$. Therefore, we have constructed an increasing sequence of subalgebras $a_k = K_k \cap \mathcal{E}(\mathcal{L}) \subset \mathcal{U}(\mathcal{G})/\mathcal{J}$ and by pulling it back into $\mathcal{U}(\mathcal{G})$, a sequence of subalgebras containing $\mathcal{J}$ and commutative modulo $\mathcal{J}$. But we wanted the $\mathcal{B}_k$'s to be of finite type modulo $\mathcal{J}$. As we were not able to show that a_k itself is of finite type, we exhibit some smaller subalgebra which has this property and which is $\mathcal{G}$-stable.

One defines a^k to be the algebra generated over a^{k-1} by all elements λ of $a_k = K_k \cap \mathcal{E}(\mathcal{L}_k)$ which can be written

$$\lambda = \sum_{i=1}^{\rho} (Y_i + \mathcal{J})\lambda_i$$

where $(Y_1, ..., Y_\rho)$ is a basis of $\mathcal{G}$ and $\lambda_i \in \mathcal{B} + \mathcal{J}$.

By standard methods of finite dimensional module over noetherian rings, one can prove that a_k is finitely generated if a_{k-1} is noetherian. Furthermore, if $X \in \mathcal{G}$, it is clear that $[X, \lambda]$ can be written in the same way, and this proves that $[\mathcal{G}, a_k] \subset a_k$. By pulling back into $\mathcal{U}(\mathcal{G})$ a set of generators of a_k, one gets the algebra $\mathcal{B}_k$, the family $(B_{N_{k-1}+1}, ..., B_{N_k})$ and the integer N_k of the proposition.

When one puts together II.3, II.4 and II.5, on gets the following

(6) *THEOREM.*(*)

Let $\mathcal{G}$ be a solvable ρ-dimensional complex Lie algebra. Then there exists

(*) Undefined notations are to be found in the addendum which follows.

a finite tree $I \subset \mathbf{N} \cup \mathbf{N}^2 \cup \dots \cup \mathbf{N}^\rho$ *and for each* $i \in I$, *a subalgebra* $\mathcal{B}_i$ *in* $\mathcal{U}(\mathcal{G})$, *subsets* $\mathcal{Z}_i$ *and* $\mathcal{J}_i$ *in* $\mathcal{B}_i$ *and* Ω_i *in* $\widehat{\mathcal{B}_i}$ *such that:*

(A) *Let* $i = (i_1, \dots, i_p) \in I$.

(1) $\mathcal{J}_i$ *is a two-sided ideal in* $\mathcal{U}(\mathcal{G})$ *and* $\mathcal{B}_i$ *is quasimaximal of S'-type on* $\mathcal{J}_i$.

(2) *One has* $i_- \in I^-$ *and*[1]

(i) $\mathcal{Z}_{i_-} \subset \mathcal{J}_{i_-} \subset \mathcal{Z}_i \subset \mathcal{J}_i$ *and* $\mathcal{B}_{i_-} \subset \mathcal{B}_i$.

(ii) $\mathcal{J}_{i-1} \neq \mathcal{Z}_i = \mathcal{J}_i \cap \mathcal{B}_{i_-}$ *if* $i_- \neq \varnothing$[1].

(iii) $\Omega_{i_-} \setminus \mathcal{X}_{i_-}$ *is dense in* $(\mathcal{B}_{i_-}/\mathcal{Z}_i)^{\wedge}$[2].

(3) Ω_i *is a constructible* $\mathcal{G}$*-stable set of S'-type characters of* $\mathcal{B}_i/\mathcal{J}_i$ *which meets* $\mathcal{X}_i$[3].

(B) *If* $j = (j_1, \dots, j_p) \in I$, *set*

$$\mathcal{X}_i^0 = \Big\{ f \in \mathcal{X}_j \,;\, \textit{for all integers } q < p \textit{ and } e \textit{ such that } (j_1, \dots, j_q, e) \in I \textit{ and } \mathcal{Z}_{j_1, \dots, j_q, e} \not\subset \mathcal{Z}_{j_1, \dots, j_q, j_{q+1}} \textit{ one has } f|\mathcal{Z}_{j_1, \dots, j_q, e} \neq 0 \Big\}$$

Then:

(1) *For all* $\varphi \in \mathcal{G}^*$, *there exist* $j \in I$ *and* $f \in \mathcal{X}_j^0 \cap \Omega_j$ *such that*

$$\varphi | b_f = \chi_f .$$

(2) *For all prime ideal* $\mathcal{L}$ *in* $\mathcal{U}(\mathcal{G})$, *there exists* $j \in I$ *such that*

(i) $\mathcal{J}_j \subset \mathcal{L}$,

(ii) $\mathcal{X}_j^0$ *is dense in* $(\mathcal{B}_j/\mathcal{L} \cap \mathcal{B}_j)^{\wedge}$,

(iii) $\mathcal{Z}_{j,k} \not\subset \mathcal{L}$ *for all integer* k *such that* $(j, k) \in I$.

Addendum to theorem 6.

(1) One sets $\mathcal{B}_\varnothing = \Gamma_\varnothing = \mathbf{C}$, $\mathcal{Z}_\varnothing = \mathcal{J}_\varnothing = \{0\}$ and $I^- = I \cup \{\varnothing\}$ $\mathbf{C} = \mathcal{B}_\varnothing$ has a unique character denoted by $id_{\mathbf{C}}$ and one sets

$$\mathcal{X}_\varnothing = \Omega_\varnothing = \{id_{\mathbf{C}}\} \text{ and } \mathcal{X}_\varnothing^0 = \varnothing.$$

For any $i = (i_1, \dots, i_p) \in \mathbf{N}^p$, set $(i, k) = (i_1, \dots, i_p, k)$

$$i_- = (i_1, \dots, i_{p-1}) \text{ and } i_- = \varnothing \text{ if } p = 1.$$

(2) Set $\mathcal{X}_i = \mathcal{X}(\mathcal{B}_i, \mathcal{J}_i)$.

(3) Clearly I is a tree as for any $i, i_- \in I$. The subalgebras $\mathcal{B}_i$ are included one into another in the same way as the indices do. Furthermore, each S'-type algebra $\mathcal{B}_i$ is defined via the elements $B_{i,1}^{\otimes}, \ldots, B_{i,N_i}^{\otimes}$ (see the remark following I.4). This set fulfils also the inclusion relation, namely if $i \in I$, one has

$$\{B_{i,1}^{\otimes}, \ldots, B_{i_-,N_{i_-}}^{\otimes}\} \subset \{B_{i,1}^{\otimes}, \ldots, B_{i,N_i}^{\otimes}\}.$$

In the definition of S'-type character, these data are in fact to be included.

The proof of this theorem is only a matter of verification.

This theorem gives the following immediate

(7) *COROLLARY.*

For all $\varphi \in \mathcal{G}^*$, there exists in $\mathcal{U}(\mathcal{G})$ a quasimaximal subalgebra $\mathcal{B}$ of S'-type on an ideal $\mathcal{J}$ and a character $f \in \mathcal{X}(\mathcal{B}, \mathcal{J})$ such that $\varphi \mid b_f = \chi_f$.

(8) *COROLLARY.*

Let $\mathcal{L}$ be a prime ideal in $\mathcal{U}(\mathcal{G})$.

There exists a two-sided ideal $\mathcal{J}$ in $\mathcal{U}(\mathcal{G})$ which is included in $\mathcal{L}$ and a quasimaximal subalgebra $\mathcal{B}$ of S'-type on $\mathcal{J}$ such that $\mathcal{X}(\mathcal{B}, \mathcal{J}) \cap (\mathcal{B}/\mathcal{L} \cap \mathcal{B})^{\wedge} \neq \varnothing$.

By II.4 (viii), this gives immediately the

(9) *PROPOSITION.*

Let $\mathcal{L}$ be a prime ideal in $\mathcal{U}(\mathcal{G})$.

There exists in $\mathcal{U}(\mathcal{G})$ a quasimaximal subalgebra $\mathcal{B}$ of S'-type on $\mathcal{L}$.

This result gives a good insight into the structure of $\mathcal{U}(\mathcal{G})/\mathcal{L}$: it is isomorphic to an algebra of differential operators on $\mathcal{B}/\mathcal{L}$. By one localisation at some element E, it can be seen as the algebra of differential operators generated by a finite set of first degree differential operators $T_1, \ldots, T_\delta$ (whose actions on the ring $\mathcal{B}/\mathcal{L}$ are linearly independent over $\mathcal{B}/\mathcal{L}$ as $\mathcal{B}$ is quasimaximal).

III. INDUCED REPRESENTATIONS.

The methods used in [4] generalize completely to the algebras given by theorem II.6. One has the following

III. 1. THEOREM.

Consider the algebras $\mathcal{B}_i$ given by II.6 and keep the notations given in II.6 and II.1. Let $i, j \in I$.

A. *S-type characters.*

Let $f, g \in (\mathcal{B}_i/\mathcal{J}_i)^\frown$ *be of S-type. Then*

(i) $\chi_f \mid [b_f, b_f] = 0$ *and* W_f *is isomorphic to* $\mathrm{Ind}(\chi_f, \mathcal{G})$.

(ii) Set $\mathcal{J}_f = \{b \in \mathcal{B}_i,\ \mathrm{ad}\, T_r \ldots \mathrm{ad}\, T_1\, b \in \ker f$ *for all* $T_1, \ldots, T_r \in \mathcal{G}\}$ *then* $\mathcal{N}_f = \{a \in \mathcal{U}(\mathcal{G}),\ a(\mathcal{B}_i \setminus \ker f) \cap \mathcal{U}(\mathcal{G})\, \mathcal{J}_f \neq \varnothing\}$.

(iii) $\chi_f \mid b_f \cap b_g = \chi_g \mid b_f \cap b_g \Rightarrow f = g$.

(iv) $\mathcal{N}_f = \mathcal{N}_g \Rightarrow \mathcal{G}_i^- f = \mathcal{G}_i^- g$[1]

(v) $f \neq g \Rightarrow W_f$ *is not isomorphic to* W_g.

(vi) W_f *is simple* $\Leftrightarrow b_f = k_f$ (i.e. $f \in \mathcal{X}(\mathcal{B}_i, \mathcal{J}_i)$).

B. *Characters in* $\mathcal{X}(\mathcal{B}_i, \mathcal{J}_i)$.

Let $f \in \mathcal{X}(\mathcal{B}_i, \mathcal{J}_i)$ *and* $\varphi \in \mathcal{G}^*$ *such that* $\varphi \mid b_f = \chi_f$. *Then*

(i) b_f *is a polarisation of* $\mathcal{G}$ *at* φ.

(ii) $\mathcal{G}\varphi = \{\psi \in \mathcal{G}^*\,;\ \exists g \in \mathcal{G}f$ *such that* $\psi \mid b_g = \chi_g\}$.

(iii) *Set* $\theta_i : \mathcal{X}_i/\mathcal{G}_i^- \longrightarrow \mathrm{Prim}\ \mathcal{U}(\mathcal{G})$

$$\mathcal{G}_i^- f \longrightarrow \mathcal{N}_f{}^{(2)}$$

Then θ_i *is a homeomorphism.*

(iv) *Let* $\mathcal{P}_i$ (*resp.* $\mathcal{P}_i^0$) *be the set of prime ideals* $\mathcal{Z}$ *in* $\mathcal{B}_i$ *such that* $\mathcal{J}_i \subset \mathcal{Z}$, $[\mathcal{G}, \mathcal{Z}] \subset \mathcal{Z}$ *and* $(\mathcal{B}_i/\mathcal{Z})^\wedge \cap \mathcal{X}_i \neq \varnothing$ (*resp.* $(\mathcal{B}_i/\mathcal{Z})^\wedge \cap \mathcal{X}_i^0 \neq \varnothing$). *Define* $\mathcal{K}_i$ *on* $\mathcal{P}_i$ *by*

$$\mathcal{K}_i(\mathcal{Z}) = \{a \in \mathcal{U}(\mathcal{G}),\ a(\mathcal{B}_i \setminus \mathcal{Z}) \cap \mathcal{U}(\mathcal{G})\mathcal{Z}\} \neq \varnothing.$$

Then $\mathcal{K}_i(\mathcal{Z})$ *is a prime ideal in* $\mathcal{U}(\mathcal{G})$ *such that* $\mathcal{K}_i(\mathcal{Z}) \cap \mathcal{B}_i = \mathcal{Z}$ *and for any prime ideal* $\mathcal{L}$ *in* $\mathcal{U}(\mathcal{G})$ *such that* $\mathcal{L} \cap \mathcal{B}_i = \mathcal{Z}$ *one has* $\mathcal{K}_i(\mathcal{Z}) = \mathcal{L}$ ($\mathcal{K}_i$ *is therefore injective*).

C. *Existence and Unicity.*

(i) *Let* $\mathcal{L}$ *be a prime ideal in* $\mathcal{U}(\mathcal{G})$. *There exists a unique* $i \in I$ *such that* $\mathcal{J}_i \subset \mathcal{L}$ *and* $(\mathcal{B}_i/\mathcal{L} \cap \mathcal{B}_i)^\wedge \cap \mathcal{X}_i^0 \neq \varnothing$.

Therefore the mappings $\mathcal{K}_i$ *define a bijection from the direct sum of the* $\mathcal{P}_i^0$*s onto the set of prime ideals of* $\mathcal{U}(\mathcal{G})$.

---

(1) $\mathcal{G}_i^-$ **is the algebraic closure of the adjoint group** $\mathcal{G}$ **acting in** $(\mathcal{B}_i/\mathcal{J}_i)^\wedge$.

(2) θ_i **exists due to** A **(iv) and (vi).**

Furthermore there exists a non void open set $w \subset (\mathcal{B}_i/\mathcal{L} \cap \mathcal{B}_i)\widehat{}$ *such that* $w \subset \mathcal{X}_i^0$, $\mathcal{L} = \bigcap_{f \in w} \mathcal{N}_f$ *and* $\mathcal{L} = \mathcal{N}_f$ *for all* $f \in w$ *if* $\mathcal{L}$ *is a primitive ideal.*

(ii) *Let* $\varphi \in \mathcal{J}^*$. *There exist a unique* $i \in I$ *and a unique* $f \in \mathcal{X}_i^0$ *such that* $\varphi | b_f = \chi_f$; *let* $\mathcal{D}(\varphi) = \mathcal{N}_f \in \mathrm{Prim}\ \mathcal{U}(\mathcal{J})$. *The mapping* φ *is constant on* $\mathcal{G}^-$*-orbits and defines a bijection from* $\mathcal{J}^*/\mathcal{G}^-$ *onto* Prim $\mathcal{U}(\mathcal{J})$.

(iii) *Set* $\Phi_i = \{\varphi \in \mathcal{J}^*, \exists\ f \in \mathcal{X}_i, \varphi | b_f = \chi_f\}$ *and* $\mathcal{D}_i = \mathcal{D} | \Phi_i$. *Then* $\mathcal{D}_i$ *defines a homeomorphism from* $\Phi_i/\mathcal{G}^-$ *into* Prim $\mathcal{U}(\mathcal{J})$.

(iv) *If* $i \neq j$, *for all* $f \in \mathcal{X}_i^0$ *and* $g \in \mathcal{X}_j^0$, $\mathcal{N}_f \neq \mathcal{N}_g$. *Therefore, the mappings* θ_i *define a bijection from the direct sum of the* $(\mathcal{X}_i^0/\mathcal{G}_i^-)$*'s onto* Prim $\mathcal{U}(\mathcal{J})$.

The proof of this theorem is very long and will not be given here.

All the material of this work comes from a

« Thèse de Doctorat d'Etat », 1976, done in the Centre Scientifique d'Orsay, Université de Paris Sud, 91405 ORSAY France. For more details please write to this address.

Some remarks are to be done.

B (i) and (ii) give the narrow connection between $\mathcal{G}$-orbits in $\mathcal{J}^*$ and $\mathcal{G}$-orbits in $\mathcal{X}_i$ (in $\mathcal{J}^*$, the dimension of the orbit is twice the dimension in $\mathcal{X}_i$).

In C, one gets all the prime ideal in $\mathcal{U}(\mathcal{J})$ and in A (ii) the intersection of the primitive ideals with $\mathcal{B}_i$ is explicitely given. In B (iv), one sees that a prime ideal is generated modulo one localization by its intersection with some $\mathcal{B}_i$.

Finally, C (ii) and C (iii) are almost the final results of the classical theory of orbits in $\mathcal{J}^*$ [5, 6, 7, 8]. The difference is that we used here non twisted induction; one gets bijectivity and piecewise homeomorphism which are the key results of the classical theory.

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