ACTA MATHEMATICA VIETNAMICA
Volume 34, Number 1, 2009, pp. 11–17

A SOLUTION FOR THE AGGREGATE PRODUCTION PLANNING PROBLEM IN A MULTI-PLANT, MULTI-PERIOD AND MULTI-PRODUCT ENVIRONMENT

LORENA PRADENAS*, CÉSAR ALVAREZ* AND JACQUES A. FERLAND**

Dedicated to Nguyen Van Hien on the occasion of his sixty-fifth birthday

Abstract. In this paper, we introduce a linear mathematical model for the Aggregate Production Planning problem for a forestry company operating several sawmills. Considering different types of raw materials, different production parameters, and a group of sawmills, the industry tries to identify the best levels of production, of subcontracting production, and of inventory to satisfy the demand for different product families in order to maximize its revenue. The linear model is solved efficiently with two linear programming software Cplex 8.1 and Xpress-Optimizer 16.10.02, and numerical results indicate the superiority of the latter.

1. Introduction

The Aggregate Production Planning (APP) model is a mid-term planning tool analyzing the relationship between the offer and the demand to determine the production levels to satisfy a demand that is not always completely known. It is useful to determine the levels of overtime production, of subcontracting production, of inventory, and of hiring/firing workforce. Using an APP model, it is also possible to determine the proper mix of resources to realise the production required (Schroeder [10]).

According to Chen et al. [2], the objective of an APP is not limited to maximize revenue of the company, but it may also be used to maximize the resource utilization, to minimize the changes in production rate or to minimize the modifications in workforce level. Hence it is important to specify correctly the objective function of an APP. In some cases, multiple objective functions are used to obtain a more realistic model for some real life situation. For instances, Baykasoglu [1] uses a Tabu search approach to deal with an APP having 4 objective functions, and Wang et al. [12] introduce a linear model having two objective functions.

Received March 30, 2008; in revised form September 10, 2008.
2000 Mathematics Subject Classification. 90B30, 90C05.
Key words and phrases. Aggregate production planning, linear programming, sawmill planning.
This research was supported by the following grants: C-13955/18 Fundacion Andes-Chile, ALFA II-0457-FA-FCD-FI-FC, DIUC-204.097.007-1.0, and NSERC 8312.

Recently, several different approaches have been proposed to deal with APP modeled as mixed integer linear programs (MILP). Jollayemi et al. [7] use a deterministic approach, Jain et al. [4], a resource based approach, and Silva et al. [11], a multi criteria approach. Gnoni et al. [3] hybridize an MILP modeling with a simulation modeling. A probabilistic linear programming approach is used by Jensen et al. [5], Jiafu et al. [6], and Wang et al. [13].

In this paper, we extend the model in Pradenas et al. [8, 9] to deal with a problem where the forestry company operates several sawmills. Hence it includes an additional level of decision related to the repartition of the overall demand and the raw material among the different sawmills. Hence we consider an APP model in a multi plants, multi products and multi periods environment where a company operating a group of sawmills has to satisfy the demand for several product families using different raw materials. The objective is to determine levels of production, of subcontracting production, and of inventory at each sawmill in order to maximize the overall profit of the company.

The paper is organized as follows. The problem is described in Section 2, and in Section 3, a model is introduced. In Section 4, six different randomly generated problems are solved using exact methods, and the numerical results indicate that the Interior Point Barrier method implemented in Express-Optimizer is more efficient than the Simplex Primal and the Simplex Dual, and also more efficient than any of these methods implemented in Cplex.

2. Problem description

In this paper, we analyse the planning problem of a forestry company operating a group of sawmills. Each sawmill has a specified production capacity, and it can be seen as an intermittent productive system type batch using four different processes: sawing, drying, sorting, and sanding. These processes generate green and dry materials that can be used in further processes generating other products. The production is characterized by the yield of the tree trunk types producing different product families according to different ways of cutting the tree trunks.

The problem can be formulated as an Aggregate Production Planning problem for a forestry company operating several sawmills located in different places, having different production capacities, and operating with different technologies. At each period of the planning horizon, the company has to determine how to allocate part of the total demand and of the raw material (tree trunks) to each sawmill in such a way that accounting for their operating capacities, the sawmills can produce the proper volumes of the different product families to satisfy the total demand. Note that it is assumed that back orders are not allowed. Accordingly, at each period, each sawmill has to determine its production level, its subcontracting production level, and its inventory level to satisfy the part of the demand assigned to it with its allocation of the raw material available.

3. Model formulation

The sawmill APP problem is formulated as a linear programming model. We assume that the company operates A sawmills to produce K product families using M tree trunk types and E cut schemes over a horizon including T periods. The following notation is used to specify the mathematical model.

Decision variables

- Y_{kta} : production level of product family k in period t at sawmill a (m^3).
- X_{meta} : quantity of tree trunk type m processed with scheme e in period t at the sawmill a (m^3).
- S_{kta} : quantity of product family k subcontracted in period t at sawmill a (m^3).
- I_{kta} : inventory level of product family k in period t at sawmill a (m^3).
- D_{kta} : demand of product family k in period t assign to sawmill a (m^3).

Parameters

- DG_{kt} : global demand of product family k in period t (m^3).
- P_{kt} : selling price of product family k in period t ($\$/m^3$).
- C_{mta} : cost per m^3 of tree trunk type m in period t for sawmill a.
- O_{kta} : production cost per m^3 of product family k in period t at sawmill a.
- SU_{kta} : subcontracting cost per m^3 of product family k in period t for sawmill a.
- CI_{kta} : inventory cost per m^3 of product family k in period t at sawmill a.
- L_{mt} : volume of tree trunks type m available in period t (m^3).
- R_{meka} : yield of tree trunk type m processed with cutting plan e to produce the family product k at sawmill a.
- V_{kta} : consumption of productive capacity for product family k in period t at sawmill a (h/m^3).
- Cap_{ta} : production capacity available in period t at sawmills a (h).

where $k = 1, \ldots, K, m = 1, \ldots, M, e = 1, \ldots, E, t = 1, \ldots, T,$ and $a = 1, \ldots, A$. The model can be summarized as follows:

$$\max Z = \sum_{t=1}^{T}\sum_{k=1}^{K} P_{kt}DG_{kt} - \sum_{t=1}^{T}\sum_{a=1}^{A}\sum_{m=1}^{M} C_{mta}\sum_{e=1}^{E} X_{meta}$$
$$- \sum_{t=1}^{T}\sum_{a=1}^{A}\sum_{k=1}^{K}(O_{kta}Y_{kta} + SU_{kta}S_{kta} + CI_{kta}I_{kta})$$

Subject to:

(1) $$\sum_{a=1}^{A} D_{kta} = DG_{kt} \quad ;\forall k,\, \forall t$$

(2) $$Y_{kta} + S_{kta} - I_{kta} \geq D_{kta} - I_{kt-1a} \quad ;\forall k,\, \forall t,\, \forall a$$

$$\sum_{a=1}^{A}\sum_{e=1}^{E} X_{meta} \leq L_{mt} \quad ;\forall m,\, \forall t \tag{3}$$

$$Y_{kta} \leq \sum_{m=1}^{M}\sum_{e=1}^{E} R_{meka}X_{meta} \quad ;\forall k,\, \forall t,\, \forall a \tag{4}$$

$$\sum_{k=1}^{K} V_{kta}Y_{kta} \leq Cap_{ta} \quad ;\forall t,\, \forall a \tag{5}$$

$$I_{k0a} \text{ is known} \quad ;\forall k,\, \forall a$$

$$X_{meta}, Y_{kta},\, S_{kta},\, I_{kta},\, D_{kta} \geq 0 \quad ;\forall k,\, \forall m,\, \forall e,\, \forall t,\, \forall a$$

The first term of the objective function corresponds to the total revenue from the sales of the products accounting for the fact that all the demand must be satisfied at each period. The second and the third terms are associated with the cost of the raw materials, and the total cost of production, subcontracting production and inventory, respectively. The constraints (1) allocate the demand among sawmills. The constraints (2) are inventory balance relations ensuring that the full demand is satisfied at each period (no back orders allowed) relying on subcontracting production if necessary. The availability of the raw material and its allocation among sawmills is specified in constraints (3). The constraints (4) are balance relations between the yield for each product family and the tree trunk types processed by the different schemes at each sawmill during each period. Finally, the capacity limiting the production at each sawmill is specified by constraints (5).

This linear programming model is deterministic because we make the assumption that the demand for each product family is known exactly. Furthermore, we assume that these demands are fully satisfied (no back orders allowed) relying on subcontracting production and inventory if necessary. In our model, we do not impose any capacity constraints on the sawmill inventory levels, but in some real context such constraints may be required inducing additional constraints in the model. Similarly, there is no limit specified on the subcontracting production levels, but there is no lost of generality since subcontracting is in general very expensive (more expensive than producing or having to handle inventory) inducing a low subcontracting production level.

We argue that we can assume that the yield R_{meka} of tree trunk type m processed with cut scheme e to produce the family product k at sawmill a is the same for all periods because the technology can be assumed to be quite stable in a mid-term planning horizon. Furthermore, the values of these yields can be specified to account for some specific situations. For instance, if the company knows that some sawmill a' is not able to reach the quality standard required for some product family k', then specifying a value 0 for the corresponding yields (i.e., $R_{mek'a'} = 0\ \forall m,\, \forall e$) induces that the sawmill a' is not producing product family k'. Similarly we can account for the technology issue where some sawmill

a' is not able to handle some tree trunk type m' by fixing the corresponding yields to 0 (i.e., $R_{m'eka'} = 0\ \forall e, \forall k$).

4. Numerical results

Six different randomly generated problems are used in our numerical experimentation. They are summarized in Table 1. They include 2 to 6 periods, 8 to 12 sawmills, 40 product families, 32 tree trunk types, and 18 cutting schemes. The largest problem includes 52992 variables and 6264 constraints.

Table 1. Test problems

Problem	K	M	E	T	A	Number of variables	Number of constraints
1	40	32	18	2	8	11776	1440
2	40	32	18	4	8	23552	2880
3	40	32	18	8	8	35328	4320
4	40	32	18	2	12	17664	2088
5	40	32	18	4	12	35328	4176
6	40	32	18	8	12	52992	6264

The problems are solved with three different exact methods (the Simplex Primal, the Simplex Dual, and the Interior Point Barrier Method) implemented in two different softwares: Cplex 8.1 and Xpress-Optimizer 16.10.02. The numerical tests have been completed using a 1.6 GHz Pentium M computer having 512 MB of Ram. The CPU time in second required by each method and each software to solve each problem, is reported in Table 2. The last column of the table includes the values of the optimal solutions.

Table 2. Test problems results

	CPU time (seconds)						
	Simplex primal		Simplex dual		Barrier method		
Problem	Cplex	Xpress	Cplex	Xpress	Cplex	Xpress	Obj. Fct.($)
1	3.48	1.66	2.59	1.77	4.20	1.30	3.358×10^7
2	9.06	8.93	16.87	8.39	62.0	3.35	6.399×10^7
3	124.61	25.51	82.79	23.31	74.26	9.32	9.627×10^7
4	6.84	2.87	5.66	3.44	4.86	1.84	5.119×10^7
5	94.07	19.14	96.76	17.79	155.55	5.28	9.744×10^7
6	373.45	47.01	336.76	52.94	443.63	16.39	1.474×10^8

The CPU time reported in Table 2 includes the time for each of the three operations required to read the data, to create the model, and to solve the problem. The numerical results indicate that Xpress-Optimizer is more efficient than Cplex to solve these problems. The large difference in efficiency is probably due to the fact that Cplex uses 57% of its CPU time to create the model while this operation requires only 0.63% of the total CPU time in the Xpress-Optimizer. Even if we eliminate the portion of the CPU time required for creating the model to compare the efficiency of the two solvers, nevertheless the adjusted results in Table 2 indicate that on the average (taken over the six problems), Xpress-Optimizer is 2.80, 1.24 and 8.60 times faster than Cplex for the Simplex primal, the Simplex dual, and the Barrier method, respectively.

In Table 3, we indicate for each problem and each software the best method and the corresponding CPU time. It is interesting to note that the Barrier method is always the best method for Xpress-Optimizer. Furthermore, considering the average solution time (after eliminating the portion for creating the model) required by Cplex even when using the best method, Xpress-Optimizer (using the Barrier method) remains 6.02 times faster. Again, the superiority of Xpress-Optimizer can be explained by the fact that it requires less time to create the model. It is also interesting to note that the Barrier method is always the best one for Xpress-Optimizer. This is consistent with the fact that the Interior Point method is known to work well when the constraint matrix is sparse. Indeed, this is the case for our test problems.

Table 3. Best results for the test problems

	Cplex		Xpress	
Problem	Method	Time [s]	Method	Time [s]
1	Simplex D.	2.59	Barrier	1.30
2	Simplex P.	9.06	Barrier	3.35
3	Barrier	74.26	Barrier	9.32
4	Barrier	4.86	Barrier	1.84
5	Simplex P.	94.07	Barrier	5.28
6	Simplex D.	336.76	Barrier	16.39

5. Conclusion

In this paper, we formulate a mathematical model for the Aggregate Production Planning problem for a forestry company operating several sawmills to determine the levels of production, of subcontracting production, and of inventory at each sawmill during each period in order to satisfy completely the demand and to maximize the profit. The numerical results indicate the superiority of the Interior Point Barrier method implemented in Xpress-Optimizer over Cplex.

References

[1] A. Baykasoglu, MOAPPS 1.0: Aggregate Production Planning using the multiple Objective Tabu Search, *Internat. J. Production Research* **39** (16) (2001), 3685–3702.

[2] Y. Chen and H. Liao, An investigation on selection of simplified aggregate production planning strategies using MADM approaches, *Internat. J. Production Research* **41** (14) (2003), 3359–3374.

[3] M. Gnoni, R. Lavagnilio, G. Mossa, G. Mummolo and A. Di Leva, Production planning of a multi-site manufacturing system by hybrid modelling: A case study from the automotive industry, *Internat. J. Production Economics* **85**(2) (2003), 251–262.

[4] A. Jain and U. Palekar, Aggregate production planning for a continuous reconfigurable manufacturing process, *Computers Oper. Res.* **32** (5) (2005), 1213–1236.

[5] H. Jensen and S. Maturana, A possibilistic decision support system for imprecise mathematical programming problems, *Internat. J. Production Economics* **77** (2) (2002), 145–158.

[6] T. Jiafu, D. Wang and R. Fung, Formulation of general possibilistic linear programming problems for complex industrial systems, *Fuzzy sets and systems* **119** (1) (2001), 41–48.

[7] J. Jolayemi and F. Olorunniwo, A deterministic model for planning production quantities in a multi-plant, multi-warehouse environment with extensible capacities, *Internat. J. Production Economics* **87** (2) (2004), 99–113.

[8] L. Pradenas, F. Peñailillo and J. Ferland, Aggregate production planning problem. A new Algorithm, *Electronic Notes in Discrete Math.* **18** (2004), 193–199.

[9] L. Pradenas, C. Alvarez and J. Ferland, *Una solución para el problema de Planificación Agregada de la Producción: Aplicación a un Aserradero Forestal*, OPTIMA Conference, Valdivia-Chile, October 2005.

[10] G. Schroeder, *Administración de Operaciones - Toma de decisiones en la función de operaciones*, Mexico: McGraw-Hill, Mexico, 1992.

[11] C. Silva, J. Figueira, J. Lisboa and S. Barman, An interactive decision support system for an aggregate production planning model based on multiple criteria mixed integer linear programming, *Omega: Internat. J. Management Sci.* **34** (2) (2006), 167–177.

[12] R. Wang and H. Fang, Aggregate production planning with multiple objectives in a fuzzy environment, *European J. Oper. Res.* **133** (3) (2001), 521–536.

[13] R. Wang and T. Liang, Applying possibility linear programming to aggregate production planning, *Internat. J. Production Economics* **98** (3) (2005), 328–341.

*University of Concepcion, Engineering Faculty,
Department of Industrial Engineering, Concepcion, Chile
E-mail address: lpradena@udec.cl
E-mail address: cealvare@udec.cl

**University of Montreal,
Department of Computer Science and Operations Research,
Montreal, Quebec, H3C 3J7
E-mail address: ferland@iro.umontreal.ca