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DIFFERENTIAL SUBORDINATION ASSOCIATED WITH LINEAR OPERATORS DEFINED FOR MULTIVALENT FUNCTIONS

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Abstract. In this paper we give certain sufficient conditions for functions defined through the Dziok-Srivastava linear operator and the multiplier transformation.

1. Introduction

Let $\mathcal{A}_p$ denote the class of all *analytic* functions $f(z)$ of the form

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k \tag{1.1}$$

$$(z \in \Delta := \{z : z \in \mathbb{C} \text{ and } |z| < 1\};\ p < k;\ p, k \in \mathbb{N} := \{1, 2, 3, \cdots\})$$

and $\mathcal{A} := \mathcal{A}_1$. Recently several authors [8, 12, 13, 16, 17, 18, 19, 25] obtained sufficient conditions associated with starlikeness in terms of the expression

$$\frac{zf'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)}.$$

In fact, Ravichandran [19] obtained the following more general result:

Theorem 1.1. [19, Theorem 3, p.44] *Let $q(z)$ be convex univalent and $0 < \alpha \leq 1$,*

$$\operatorname{Re}\left\{\frac{1-\alpha}{\alpha} + 2q(z) + \left(1 + \frac{zq''(z)}{q'(z)}\right)\right\} > 0.$$

If $f \in \mathcal{A}$ satisfies

$$\frac{zf'(z)}{f(z)} + \alpha \frac{z^2 f''(z)}{f(z)} \prec (1-\alpha)q(z) + \alpha q^2(z) + \alpha z q'(z),$$

then $\frac{zf'(z)}{f(z)} \prec q(z)$ and $q(z)$ is the best dominant.

Also the following extension of a result of Darus and Frasin [6] was obtained:

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Theorem 1.2. [19, Theorem 4, p.48] *Let $q(z)$ be analytic in Δ, $q(0) = 1$ and $h(z) = zq'(z)/q(z)$ be starlike univalent in Δ. If $f \in \mathcal{A}$ satisfies*

$$\frac{(zf(z))''}{f'(z)} - 2\frac{zf'(z)}{f(z)} \prec h(z),$$

then

$$\frac{z^2 f'(z)}{f^2(z)} \prec q(z).$$

The dominant $q(z)$ is the best dominant.

In the present paper, the authors present extension of the above two theorems for functions defined through Dziok-Srivastava linear operator and the multiplier transformation on the space of multivalent functions $\mathcal{A}_p$.

2. Preliminaries

For two analytic functions $f(z)$ given by (1.1) and

$$g(z) = z^p + \sum_{k=p+1}^{\infty} b_k z^k,$$

their Hadamard product (or convolution) is the function $(f * g)(z)$ defined by

$$(f * g)(z) := z^p + \sum_{k=p+1}^{\infty} a_k b_k z^k.$$

For $\alpha_j \in \mathbb{C}$ $(j = 1, 2, \dots, l)$ and $\beta_j \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$, $j = 1, 2, \dots, m$, the *generalized hypergeometric function* ${}_lF_m(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m)$ is defined by the infinite series

$${}_lF_m(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m) := \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \dots (\alpha_l)_n}{(\beta_1)_n \dots (\beta_m)_n} \frac{z^n}{n!}$$

$$(l \leq m + 1;\ l, m \in \mathbb{N}_0 := \mathbb{N} \cup \{0\})$$

where $(a)_n$ is the Pochhammer symbol defined by

$$(a)_n := \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} 1 & \text{if } n = 0; \\ a(a+1)(a+2)\dots(a+n-1) & \text{if } n \in \mathbb{N}. \end{cases}$$

Corresponding to the function

$$h_p(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m) := z^p\ {}_lF_m(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m),$$

the Dziok-Srivastava operator [5] (see also [23]) $H_p^{(l,m)}(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m)$ is defined by the Hadamard product

$$\begin{aligned} H_p^{(l,m)}(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m) f(z) &:= h_p(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m) * f(z) \\ &= z^p + \sum_{n=p+1}^{\infty} \frac{(\alpha_1)_{n-p} \dots (\alpha_l)_{n-p}}{(\beta_1)_{n-p} \dots (\beta_m)_{n-p}} \frac{a_n z^n}{(n-p)!}. \end{aligned} \tag{2.1}$$

It is well known [5] that

(2.2)
$$\alpha_1 H_p^{(l,m)}(\alpha_1+1,\dots,\alpha_l;\beta_1,\dots,\beta_m)f(z) = z[H_p^{(l,m)}(\alpha_1,\dots,\alpha_l;\beta_1,\dots,\beta_m)f(z)]' + (\alpha_1-p)H_p^{(l,m)}(\alpha_1,\dots,\alpha_l;\beta_1,\dots,\beta_m)f(z).$$

The Dziok-Srivastava linear operator includes special cases the Hohlov linear operator [7], the Carlson-Shaffer linear operator [2], the Ruscheweyh derivative operator [20], the generalized Bernardi-Libera-Livingston linear integral operator (*cf.* [1], [9], [10]) and the Srivastava-Owa fractional derivative operators (*cf.* [15], [14]).

Motivated by the multiplier transformation on $\mathcal{A}$, we define the operator $I_p(n,\lambda)$ on $\mathcal{A}_p$ by the following infinite series

$$I_p(n,\lambda)f(z) := z^p + \sum_{k=p+1}^{\infty}\left(\frac{k+\lambda}{p+\lambda}\right)^n a_k z^k. \tag{2.3}$$

A straight forward calculation shows that

$$(p+\lambda)I_p(n+1,\lambda)f(z) = z[I_p(n,\lambda)f(z)]' + \lambda I_p(n,\lambda)f(z). \tag{2.4}$$

The operator $I_p(n,\lambda)$ is closely related to the Sălăgean derivative operators [21]. The operator $I_\lambda^n := I_1(n,\lambda)$ was recently studied by Cho and Srivastava [3] and Cho and Kim [4]. The operator $I_n := I_1(n,1)$ was studied by Uralegaddi and Somanatha [24].

We shall need the following lemma due to Miller and Mocanu

Lemma 2.1. [11, Theorem 3.4h, p.132] *Let $q(z)$ be univalent in the unit disk Δ. Let ϑ and φ be analytic in a domain D containing $q(\Delta)$ with $\varphi(w)\neq 0$ when $w\in q(\Delta)$. Set*

$$Q(z) := zq'(z)\varphi(q(z)),$$
$$h(z) := \vartheta(q(z)) + Q(z).$$

Suppose that either

(1) *$h(z)$ is convex, or*

(2) *$Q(z)$ is starlike univalent in Δ.*

In addition, assume that

$$\Re\frac{zh'(z)}{Q(z)} > 0 \text{ for } z\in\Delta.$$

If $p(z)$ is analytic with $p(0)=q(0)$, $p(\Delta)\subseteq D$ and

$$\vartheta(p(z)) + zp'(z)\varphi(p(z)) \prec \vartheta(q(z)) + zq'(z)\varphi(q(z)), \tag{2.5}$$

then

$$p(z)\prec q(z) \tag{2.6}$$

and $q(z)$ is the best dominant.

The following result which is a special case of Lemma 2.1 is also useful.

Lemma 2.2. [11, Corollary 3.4h.1, p.135] *Let $q(z)$ be univalent in Δ and let $\varphi(z)$ be analytic in a domain containing $q(\Delta)$. If $zq'(z)/\varphi(q(z))$ is starlike, then*

$$z\psi'(z)\varphi(\psi(z)) \prec zq'(z)\varphi(q(z)) \quad (z \in \Delta),$$

then $\psi(z) \prec q(z)$ and $q(z)$ is the best dominant.

By making use of Lemma 2.1, we prove the following

Lemma 2.3. *If $p(z)$ and $q(z)$ are analytic in Δ, $q(z)$ is convex univalent, α, β and γ are complex and $\gamma \neq 0$. Further assume that*

$$\Re\left\{\frac{\alpha}{\gamma} + \frac{2\beta}{\gamma}q(z) + \left(1 + \frac{zq''(z)}{q'(z)}\right)\right\} > 0.$$

If $p(z) = 1 + cz + \cdots$ is analytic in Δ and satisfies

$$\alpha p(z) + \beta p^2(z) + \gamma zp'(z) \prec \alpha q(z) + \beta q^2(z) + \gamma zq'(z),$$

then $p(z) \prec q(z)$ and $q(z)$ is the best dominant.

Proof. Let $\vartheta(w) := \alpha w + \beta w^2$ and $\varphi(w) := \gamma$. Then clearly $\vartheta(w)$ and $\varphi(w)$ are analytic in $\mathbb{C}$ and $\varphi(w) \neq 0$. Also let

$$Q(z) = zq'(z)\varphi(q(z)) = \gamma zq'(z)$$

and

$$h(z) = \vartheta(q(z)) + Q(z) = \alpha q(z) + \beta q^2(z) + \gamma zq'(z).$$

Since $q(z)$ is convex univalent, $zq'(z)$ is starlike univalent. Therefore $Q(z)$ is starlike univalent in Δ, and

$$\Re\frac{zh'(z)}{Q(z)} = \Re\left\{\frac{\alpha}{\gamma} + \frac{2\beta}{\gamma}q(z) + \left(1 + \frac{zq''(z)}{q'(z)}\right)\right\} > 0$$

for $z \in \Delta$. Hence the result follows from Lemma 2.1. $\square$

Remark. Note that the condition $\Re\left\{\frac{\alpha}{\gamma} + \frac{2\beta}{\gamma}q(z) + \left(1 + \frac{zq''(z)}{q'(z)}\right)\right\} > 0$ is satisfied by any convex function that maps Δ onto a convex region in the right-half plane when $\alpha > 0$, $\beta > 0$ and $\gamma > 0$.

3. Sufficient conditions involving Dziok-Srivastava linear operator

To make the notation simple, we write

$$H_p^{l,m}(\alpha_1)f(z) := H_p^{(l,m)}(\alpha_1, \ldots, \alpha_l; \beta_1, \ldots, \beta_m)f(z).$$

By making use of Lemma 2.3 we first prove the following

Theorem 3.1. *Let $q(z)$ be convex univalent, $\alpha \neq 0$. Further assume that*

$$\Re\left\{\frac{1 + \alpha_1(1-\alpha)}{\alpha} + \frac{2(1 + \alpha_1(1-\alpha))}{\alpha\alpha_1}q(z) + \left(1 + \frac{zq''(z)}{q'(z)}\right)\right\} > 0.$$

If $f(z) \in \mathcal{A}_p$ *satisfies*

$$\frac{H_p^{l,m}(\alpha_1+1)f(z)}{H_p^{l,m}(\alpha_1)f(z)}\left\{1-\alpha+\alpha\frac{H_p^{l,m}(\alpha_1+2)f(z)}{H_p^{l,m}(\alpha_1+1)f(z)}\right\}$$

$$\prec \frac{1+\alpha_1(1-\alpha)}{1+\alpha_1}q(z)+\frac{\alpha\alpha_1}{1+\alpha_1}q^2(z)+\frac{\alpha}{1+\alpha_1}zq'(z) \tag{3.1}$$

then

$$\frac{H_p^{l,m}(\alpha_1+1)f(z)}{H_p^{l,m}(\alpha_1)f(z)} \prec q(z) \tag{3.2}$$

and q(z) is the best dominant.

Proof. Define the function $\psi(z)$ by

$$\psi(z) := \frac{H_p^{l,m}(\alpha_1+1)f(z)}{H_p^{l,m}(\alpha_1)f(z)}. \tag{3.3}$$

By a simple computation from (3.3) we get

$$\frac{z\psi'(z)}{\psi(z)} = \frac{z[H_p^{l,m}(\alpha_1+1)f(z)]'}{H_p^{l,m}(\alpha_1+1)f(z)} - \frac{z[H_p^{l,m}(\alpha_1)f(z)]'}{H_p^{l,m}(\alpha_1)f(z)}. \tag{3.4}$$

By making use of (2.2) in the equation (3.4), we obtain

$$\frac{z\psi'(z)}{\psi(z)} = (\alpha_1+1)\frac{H_p^{l,m}(\alpha_1+2)f(z)}{H_p^{l,m}(\alpha_1+1)f(z)} - \alpha_1\frac{H_p^{l,m}(\alpha_1+1)f(z)}{H_p^{l,m}(\alpha_1)f(z)} - 1. \tag{3.5}$$

Using (3.3) in (3.5), we get

$$\frac{H_p^{l,m}(\alpha_1+2)f(z)}{H_p^{l,m}(\alpha_1+1)f(z)} = \frac{1}{1+\alpha_1}\left[\frac{z\psi'(z)}{\psi(z)}+\alpha_1\psi(z)+1\right]. \tag{3.6}$$

Therefore we have from (3.6),

$$\frac{H_p^{l,m}(\alpha_1+1)f(z)}{H_p^{l,m}(\alpha_1)f(z)}\left\{1-\alpha+\alpha\frac{H_p^{l,m}(\alpha_1+2)f(z)}{H_p^{l,m}(\alpha_1+1)f(z)}\right\}$$

$$= \frac{1+\alpha_1(1-\alpha)}{1+\alpha_1}\psi(z)+\frac{\alpha\alpha_1}{1+\alpha_1}\psi^2(z)+\frac{\alpha}{1+\alpha_1}z\psi'(z). \tag{3.7}$$

In view of the equation (3.7), the subordination (3.1) becomes

$$[1+\alpha_1(1-\alpha)]\psi(z)+\alpha\alpha_1\psi^2(z)+\alpha z\psi'(z) \prec [1+\alpha_1(1-\alpha)]q(z) + \alpha\alpha_1 q^2(z)+\alpha zq'(z)$$

and the result now follows by an application of Lemma 2.3. ☐

By making use of Lemma 2.2, we now prove the following

Theorem 3.2. *Let $q(z)$ be univalent in Δ, $q(0) = 1$. Let $zq'(z)/q(z)$ be starlike univalent in Δ. If $f(z) \in \mathcal{A}_p$ satisfies*

$$(\alpha_1 + 1)\frac{H_p^{l,m}(\alpha_1 + 2)f(z)}{H_p^{l,m}(\alpha_1 + 1)f(z)} - \alpha\alpha_1\frac{H_p^{l,m}(\alpha_1 + 1)f(z)}{H_p^{l,m}(\alpha_1)f(z)}$$

$$\prec \frac{zq'(z)}{q(z)} + 1 - \alpha_1 - \alpha\alpha_1 \tag{3.8}$$

then

$$\frac{z^{p(\alpha-1)}H_p^{l,m}(\alpha_1 + 1)f(z)}{(H_p^{l,m}(\alpha_1)f(z))^\alpha} \prec q(z) \tag{3.9}$$

and q(z) is the best dominant.

Proof. Define the function $\psi(z)$ by

$$\psi(z) := \frac{z^{p(\alpha-1)}H_p^{l,m}(\alpha_1 + 1)f(z)}{(H_p^{l,m}(\alpha_1)f(z))^\alpha}. \tag{3.10}$$

By a simple computation from (3.10) we get

$$\frac{z\psi'(z)}{\psi(z)} = \alpha - 1 + \frac{z[H_p^{l,m}(\alpha_1 + 1)f(z)]'}{H_p^{l,m}(\alpha_1 + 1)f(z)} - \alpha\frac{z[H_p^{l,m}(\alpha_1)f(z)]'}{H_p^{l,m}(\alpha_1)f(z)}. \tag{3.11}$$

By making use of (2.2) in the equation (3.11), we obtain

$$(\alpha_1 + 1)\frac{H_p^{l,m}(\alpha_1 + 2)f(z)}{H_p^{l,m}(\alpha_1 + 1)f(z)} - \alpha\alpha_1\frac{H_p^{l,m}(\alpha_1 + 1)f(z)}{H_p^{l,m}(\alpha_1)f(z)}$$

$$= \frac{z\psi'(z)}{\psi(z)} + 1 - \alpha_1 - \alpha\alpha_1. \tag{3.12}$$

In view of the equation (3.12), the subordination (3.8) becomes

$$\frac{z\psi'(z)}{\psi(z)} \prec \frac{zq'(z)}{q(z)}$$

and the result now follows by an application of Lemma 2.2. ∎

4. Sufficient conditions involving multiplier transform

By making use of Lemma 2.3, we prove the following

Theorem 4.1. *Let $q(z)$ be convex univalent, $\alpha \neq 0$. Further assume that*

$$\Re\left\{\frac{(1-\alpha)(p+\lambda)}{\alpha} + 2(p+\lambda)q(z) + \left(1 + \frac{zq''(z)}{q'(z)}\right)\right\} > 0.$$

If $f(z) \in \mathcal{A}_p$ satisfies

$$\frac{I_p(n+1,\lambda)f(z)}{I_p(n,\lambda)f(z)}\left\{1 - \alpha + \alpha\frac{I_p(n+2,\lambda)f(z)}{I_p(n+1,\lambda)f(z)}\right\}$$

$$\prec (1-\alpha)q(z) + \alpha q^2(z) + \frac{\alpha}{p+\lambda}zq'(z), \tag{4.1}$$

then

$$\frac{I_p(n+1,\lambda)f(z)}{I_p(n,\lambda)f(z)} \prec q(z) \tag{4.2}$$

and q(z) is the best dominant.

Proof. Define the function $\psi(z)$ by

$$\psi(z) := \frac{I_p(n+1,\lambda)f(z)}{I_p(n,\lambda)f(z)}. \tag{4.3}$$

By a simple computation from (4.3) we get

$$\frac{z\psi'(z)}{\psi(z)} = \frac{z[I_p(n+1,\lambda)f(z)]'}{I_p(n+1,\lambda)f(z)} - \frac{z[I_p(n,\lambda)f(z)]'}{I_p(n,\lambda)f(z)}. \tag{4.4}$$

By making use of (2.4) in the equation (4.4), we obtain

$$\frac{z\psi'(z)}{\psi(z)} = (p+\lambda)\left[\frac{I_p(n+2,\lambda)f(z)}{I_p(n+1,\lambda)f(z)} - \frac{I_p(n+1,\lambda)f(z)}{I_p(n,\lambda)f(z)}\right]. \tag{4.5}$$

Using (4.3) in (4.5), we get

$$\frac{I_p(n+2,\lambda)f(z)}{I_p(n+1,\lambda)f(z)} = \frac{1}{p+\lambda}\left[\frac{z\psi'(z)}{\psi(z)} + (p+\lambda)\psi(z)\right]. \tag{4.6}$$

Therefore we have from (4.6),

$$\begin{aligned}\frac{I_p(n+1,\lambda)f(z)}{I_p(n,\lambda)f(z)}\left\{1-\alpha+\alpha\frac{I_p(n+2,\lambda)f(z)}{I_p(n+1,\lambda)f(z)}\right\} =&(1-\alpha)\psi(z)\\ &+\alpha\psi^2(z)+\frac{\alpha}{p+\lambda}z\psi'(z).\end{aligned} \tag{4.7}$$

In view of the equation (4.7), the subordination (4.1) becomes

$$(1-\alpha)\psi(z)+\alpha\psi^2(z)+\frac{\alpha}{p+\lambda}z\psi'(z)$$

$$\prec (1-\alpha)q(z)+\alpha q^2(z)+\frac{\alpha}{p+\lambda}zq'(z)$$

and the result now follows by an application of Lemma 2.3. □

By making use of Lemma 2.2, we now prove the following

Theorem 4.2. *Let $q(z)$ be univalent in Δ, $q(0)=1$. Let $zq'(z)/q(z)$ be starlike univalent in Δ. If $f(z)\in\mathcal{A}_p$ satisfies*

$$\frac{I_p(n+2,\lambda)f(z)}{I_p(n+1,\lambda)f(z)} - \alpha\frac{I_p(n+1,\lambda)f(z)}{I_p(n,\lambda)f(z)} \prec \frac{1}{p+\lambda}\frac{zq'(z)}{q(z)}+1-\alpha,$$

then

$$\frac{z^{p(\alpha-1)}I_p(n+1,\lambda)f(z)}{(I_p(n,\lambda)f(z))^{\alpha}} \prec q(z) \tag{4.8}$$

and q(z) is the best dominant.

Proof. Define the function $\psi(z)$ by

$$\psi(z) := \frac{z^{p(\alpha-1)} I_p(n+1,\lambda) f(z)}{(I_p(n,\lambda)f(z))^\alpha}. \tag{4.9}$$

By a simple computation from (4.9) we get

$$\frac{z\psi'(z)}{\psi(z)} = \alpha - 1 + \frac{z[I_p(n+1,\lambda)f(z)]'}{I_p(n+1,\lambda)f(z)} - \alpha\frac{z[I_p(n,\lambda)f(z)]'}{I_p(n,\lambda)f(z)}. \tag{4.10}$$

By making use of (2.4) in the equation (4.10), we obtain

$$\frac{I_p(n+2,\lambda)f(z)}{I_p(n+1,\lambda)f(z)} - \alpha\frac{I_p(n+1,\lambda)f(z)}{I_p(n,\lambda)f(z)} = \frac{1}{p+\lambda}\frac{z\psi'(z)}{\psi(z)} + 1 - \alpha. \tag{4.11}$$

In view of the equation (4.11), the subordination (4.2) becomes

$$\frac{z\psi'(z)}{\psi(z)} \prec \frac{zq'(z)}{q(z)}$$

and the result now follows by an application of Lemma 2.2. □

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