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PERMANENCE AND POSITIVE BOUNDED SOLUTIONS OF KOLMOGOROV COMPETING SPECIES SYSTEM

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Abstract. We study the Kolmogorov equation for n-species. Under certain conditions, it is shown that the equation is permanent and there exists a solution defined on whole R whose components are bounded above and below by positive constants.

1. Introduction

Consider the n-species Kolmogorov-type nonautononous differential system

$$\dot{x}_i = x_i f_i(t, x_1, ..., x_n),\ i = 1, ..., n, \tag{1.1}$$

where $f_i : R \times R^n_+ \to R$ is uniformly continuous on $R \times R^n_+$ ($R^n_+ := \{x \in R^n : x_i \geq 0,\ i = 1, ..., n\}$).

A special case of (1.1) is the Lotka-Volterra-type system

$$\dot{x}_i = x_i \Big[b_i(t) - \sum_{j=1}^{n} a_{ij}(t) x_j \Big], \quad i = 1, ..., n, \tag{1.2}$$

where the functions b_i, $a_{ij} : R \to R$ are bounded and continuous.

A fundamental ecological question associated with the study of multi-species population interactions is the long term coexistence of the involved populations. Such questions arise also in many other situations (see [6]). Mathematically, this is equivalent to the so-called permanence of the populations. We recall that *the system* (1.1) *is permanent if there exist positive constants* m, M $(m \leq M)$ *such that any noncontinuable solution* $x(.)$ *of* (1.1) *with* $x(t_0) \in \mathrm{int}(R^n_+)$*-the interior of* R^n_+*, is defined on* $[t_0, +\infty)$ *and the following condition is satisfied*

$$m \leq \liminf_{t \to +\infty} x_i(t) \leq \limsup_{t \to +\infty} x_i(t) \leq M, \quad i = 1, ..., n. \tag{1.3}$$

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Zanolin [12] and Zhao [13] studied the permanence and the existence of positive periodic solutions of the systems (1.1) and (1.2) in the periodic case. Some results on the existence of periodic and almost periodic solution and the stability of the system (1.2) in the periodic and almost periodic cases were given [2, 3, 4, 9]. In general, if system (1.1) is not periodic, then it may have positive bounded solutions (defined on whole R). For system (1.2) in the case $n = 2$, some sufficient conditions for the existence of a positive bounded solution were given in [1, 10].

In this paper we present a result on the permanence and the existence of positive and bounded solutions of the system (1.1) and (1.2).

2. Main results

We introduce the following hypotheses:

(H_1) There exists a positive number α such that $D_{x_i} f_i(t, x_1, \ldots, x_n) \leq -\alpha$, $(t, x) \in R \times R_+^n$, $i = 1, \ldots, n$, where D_{x_i} is any Dini derivative in x_i,

(H_2) $\inf\limits_{t \in R} f_i(t, 0) > 0$ and $f_i(t, x)$ is bounded on the sets of the form $R \times S$, where S is any compact subset of R_+^n $(i = 1, \ldots, n)$.

(H_3) $f_i(t, x_1, \ldots, x_n)$ is decreasing in x_j $(i, j = 1, \ldots, n)$.

From now on, let $R_+ := [0, +\infty)$ and $\theta := (0, \ldots, 0) \in R^n$. For $x, z \in R^n$ we set $x \leq z$ if $x_i \leq z_i$, $i = 1, \ldots, n$. We denote by $\mathcal{C}^+$ the set of continuous functions from R into R which are bounded above and below by positive constants.

The following lemmas were proved in [11].

Lemma 1. *Suppose that $g : R \times R_+ \to R$ is continuous such that*

(G_1) $g(., 0) \in \mathcal{C}^+$,

(G_2) *There exists a positive α such that $D_y g(t, y) \leq -\alpha$ for all $(t, y) \in R \times R_+$,*

(G_3) *There exists $\varepsilon > 0$ such that $g(t, x)$ is uniformly continuous on $R \times [0, \varepsilon]$.*

Then the problem

$$\dot{y} = y g(t, y), \quad y(.) \in \mathcal{C}^+, \tag{2.1}$$

has a unique solution $y^0(.)$. Moreover, we have $\lim\limits_{t \to +\infty} |y^0(t) - y(t)| = 0$ for any solution $y(.)$ of the equation in (2.1) *with $y(t_0) > 0$.*

Lemma 2. *Suppose that $g : R \times R_+ \to R$ is almost periodic in t uniformly for $y \in R_+$ such that*

(G_1^*) $\lim\limits_{\omega\to\infty} \dfrac{1}{\omega}\displaystyle\int_0^\omega g(t,0)dt > 0,$

(G_2^*) *There exists a positive* α *such that* $D_y g(t,y) \le -\alpha$ *for all* $(t,y) \in R \times R_+$.

Then problem (2.1) *has a unique solution* $y^0(.)$. *Moreover,* $y^0(.)$ *is almost periodic and we have* $\lim\limits_{t\to+\infty} |y^0(t) - y(t)| = 0$ *for any solution* $y(.)$ *of the equation in* (2.1) *with* $y(t_0) > 0$.

By Lemma 1, for each $i = 1, \ldots, n$ the problem

$$(2.2_i) \qquad \dot{x}_i = x_i f_i(t, 0, ..., 0, x_i, 0, ..., 0), \quad x_i(.) \in \mathcal{C}^+,$$

has a unique solution, say $X_i^0(.)$, which is bounded above and below by positive constants. From now on we set $X^0(.) = (X_1^0(.), ..., X_n^0(.))$.

Our main results are the following:

Theorem 1. *Assume* (H_1), (H_2), (H_3). *If*

(H_4) $\inf\limits_{t\in R} f_i(t, X_1^0(t), ..., X_{i-1}^0(t), 0, X_{i+1}^0(t), ..., X_n^0(t)) > 0,\ i = 1, ..., n,$

then system (1.1) *is permanent and it has at least one solution* $x^*(.) = (x_1^*(.), ..., x_n^*(.))$ *defined on whole* R, *whose components are bounded above and below by positive constants.*

Theorem 2. *Suppose that* $f_i(t,x)$ $(i = 1, ..., n)$ *is almost periodic in* t *uniformly for* $x \in R_+^n$ *and satisfies* (H_1), (H_3) *and the following conditions for* $i = 1, \ldots, n$,

(H_2^*) $\lim\limits_{\omega\to\infty} \dfrac{1}{\omega}\displaystyle\int_0^\omega f_i(t,0) > 0,$

(H_4^*) $\lim\limits_{\omega\to\infty} \dfrac{1}{\omega}\displaystyle\int_0^\omega f_i(t, X_1^0(t), ..., X_{i-1}^0(t), 0, X_{i+1}^0(t), ..., X_n^0(t))dt > 0.$

Then system (1.1) *is permanent and it has at least one solution* $x^*(.) = (x_1^*(.), ..., x_n^*(.))$ *defined on whole* R, *whose components are bounded above and below by positive constants.*

Applying Theorem 2 to system (1.2) we get the following

Corollary 1. *Suppose that* $b_i(t)$, $a_{ij}(t)$ $(i,j = 1, ..., n)$ *are almost periodic functions such that*

(L_1) $\inf_{t\in R} a_{ii}(t) > 0$, $\lim_{\omega\to\infty} \frac{1}{\omega}\int_0^\omega b_i(t)dt > 0$, $a_{ij}(t) \geq 0$, $(i,j = 1,...,n,$ $t \in R)$.

If

(L_2) $\lim_{\omega\to\infty} \frac{1}{\omega}\int_0^\omega \Big[b_i(t) - \sum_{j=1,j\neq i}^{n} a_{ij}(t)X_j^0(t)\Big]dt > 0$, $i = 1,...,n$,

where $X_j^0(.)$ *is the unique almost periodic solution of the following problem*

$$\dot{x}_j = x_j[b_j(t) - a_{jj}(t)x_j], \quad x_j(.) \in \mathcal{C}^+,$$

then system (1.2) *is permanent and it has at least one solution defined on whole* R, *whose components are bounded above and below by positive constants.*

It is clear that

$$\sup_{t\in R} X_j^0(t) \leq K_j := \sup_{t\in R} b_j(t)/a_{jj}(t).$$

Therefore (L_2) holds if

(L_2') $\lim_{\omega\to\infty} \frac{1}{\omega}\int_0^\omega \Big[b_i(t) - \sum_{j=1,j\neq i}^{n} K_j a_{ij}(t)\Big]dt > 0$, $i = 1,...,n$.

Thus we have the following corollary.

Corollary 2. *Suppose that the functions* $b_i(t)$, $a_{ij}(t)$ $(i,j = 1,...,n)$ *are almost periodic and* (L_1), (L_2') *hold. Then the assertion of Corollary 1 is valid.*

Let $\mathcal{M}$ be the space of continuous functions from R into R^n equipped with the topology of uniform convergence on compact subsets of R. It is well-known that $\mathcal{M}$ is a Frechet space. Let

$$\mathcal{M}_1 := \{p \in \mathcal{M} : \theta \leq p(t) \leq X^0(t),\ t \in R\}.$$

By Lemma 1 and the hypotheses (H_1), (H_2), (H_3), for each $p \in \mathcal{M}_1$ the following system of n uncouple differential equations

$$\dot{z}_i = z_i f_i(t, p_1(t), ..., p_{i-1}(t), z_i, p_{i+1}(t), ..., p_n(t)), \quad i = 1, ...n \tag{2.3}$$

has a unique solution $z(p)(.) \in \mathcal{M}$ whose components are bounded above and below by positive constants. Hence, we can introduce the operator

$$\begin{aligned} &T : \mathcal{M}_1 \to \mathcal{M} \\ &p \mapsto T(p) = z(p)(.). \end{aligned}$$

Clearly, $p(.)$ is a solution in $\mathcal{M}_1$ of (1.1) if and only if it is a fixed point of T. We shall apply an extension of Schauder's fixed point theorem, namely the Tychonov fixed point theorem, for proving the existence of a fixed point for the operator T.

Theorem 3 (Tychonov). *Let $\mathcal{X}$ be locally convex and Hausdorff, $C \subset \mathcal{X}$ closed convex, $F : C \to C$ continuous and $F(C)$ precompact. Then F has a fixed point.*

Moreover, Ascoli's theorem is also used in our proof.

Theorem 4 (Ascoli). *Let $\mathcal{X}$ be a topological space which is locally compact Hausdorff, $\mathcal{Y}$ a metric space, and $\mathcal{C}(\mathcal{X}, \mathcal{Y})$ the space of continuous functions from $\mathcal{X}$ into $\mathcal{Y}$. Consider $\mathcal{C}(\mathcal{X}, \mathcal{Y})$ with the topology of uniform convergence on compact subsets of $\mathcal{X}$. A subset $\mathcal{F}$ of $\mathcal{C}(\mathcal{X}, \mathcal{Y})$ has compact closure if and only if it is equicontinuous and the subset $\mathcal{F}_x = \{h(x) | h \in \mathcal{F}\}$ of $\mathcal{Y}$ has compact closure for each $x \in \mathcal{X}$.*

Note that in this paper we do not assume the uniqueness of solutions to the Cauchy problem for system (1.1). The proof of Theorem 1 is based on the following lemmas.

Lemma 3. *If $p^1, p^2 \in \mathcal{M}_1$, $p^1(t) \le p^2(t)$ $(t \in R)$, then $T(p^1)(t) \ge T(p^2)(t)$.*

Proof. Suppose the assertion of the lemma is false, i.e., there exist $i \in \{1, ..., n\}$ and $t_0 \in R$ such that $(T(p^1))_i(t_0) < (T(p^2))_i(t_0)$. Put $z^1 = T(p^1)$, $z^2 = T(p^2)$ and $A(t) = \ln z_i^1(t) - \ln z_i^2(t)$. Note that $A(t_0) < 0$ and $z^k(t)$ $(k = 1, 2)$ satisfies the system (2.3) where p is replaced by p^k, i.e.,

$$(2.3_k) \quad \dot{z}_i^k = z_i^k f_i(t, p_1(t), ..., p_{i-1}(t), z_i^k, p_{i+1}(t), ..., p_n(t)), \quad i = 1, ..., n.$$

By (H_3), we have $\dot{A}(t_0) > 0$.
Claim. $\dot{A}(t) > 0$ for all $t \in (-\infty, t_0]$.

Suppose the claim were false, i.e., there exists $t_1 < t_0$ such that $\dot{A}(t) > 0$ for all $t \in (t_1, t_0]$ and $\dot{A}(t_1) = 0$. This implies that $A(t)$ is strictly increasing on $[t_1, t_0]$. Thus $z_i^1(t_1) < z_i^2(t_1)$ and, consequently, $\dot{A}(t_1) > 0$. This is a contradiction.

It follows from the claim that $z_i^1(t) < z_i^2(t)$ for all $t \in (-\infty, t_0]$. By (2.3_k), (H_1), (H_2) we have

$$\begin{aligned}\dot{A}(t) &= f_i(t, p_1^1(t), ..., p_{i-1}^1, z_i^1(t), p_{i+1}^1(t), ..., p_n^1(t)) \\ &\quad - f_i(t, p_1^2(t), ..., p_{i-1}^2, z_i^2(t), p_{i+1}^2(t), ..., p_n^2(t)) \\ &\geq \alpha[z_i^2(t) - z_i^1(t)] > 0, \ t \in (-\infty, t_0].\end{aligned}$$

Since $z_i^1(.)$, $z_i^2(.) \in \mathcal{C}^+$, there exists $\beta > 0$ such that

$$0 < \int_{\omega}^{t_0} \dot{A}(t)dt = \ln \frac{z_i^1(t_0) z_i^2(\omega)}{z_i^2(t_0) z_i^1(\omega)} < \beta \quad \text{for all} \ \ \omega \leq t_0.$$

Thus

$$0 < \int_{-\infty}^{t_0} \alpha[z_i^2(t) - z_i^1(t)]dt < +\infty.$$

Since $\dot{z}_i^1(t)$, $\dot{z}_i^2(t)$ are bounded, we have

$$\lim_{t \to -\infty} [z_i^2(t) - z_i^1(t)] = 0$$

and, consequently, $\lim\limits_{t \to -\infty} A(t) = 0$. This implies that

$$\int_{-\infty}^{t_0} \dot{A}(t)dt = A(t_0) < 0,$$

which contradicts

$$\int_{-\infty}^{t_0} \dot{A}(t)dt \geq \alpha \int_{-\infty}^{t_0} [z_i^2(t) - z_i^1(t)]dt > 0.$$

The lemma is proved. $\square$

By Lemma 3, we have $T(X^0)(t) \leq T(\theta)(t) = X^0(t), \ t \in R$. Let us set

$$\mathcal{M}_2 = \{P \in \mathcal{M} : T(X^0)(t) \leq p(t) \leq X^0(t), \ t \in R\},$$

$$\delta = \inf_{t\in R,\ 1\le i\le n} T(X^0)_i(t),$$
$$\Delta = \sup_{t\in R,\ 1\le i\le n} X_i^0(t).$$

It is clear that $0 < \delta \le \Delta < +\infty$. It follows from (H_2) that

$$0 < L := \sup_{(t,x)\in R\times[\delta,\Delta],\ 1\le i\le n} |f_i(t,x)| < +\infty.$$

Let us set

$$\mathcal{M}_3 = \{p \in \mathcal{M}_2 : |p_i(t) - p_i(t')| \le L|t-t'|, i = 1, ..., n,\ t \in R\}.$$

It is easily seen that $\mathcal{M}_3$ is a closed convex subset of $\mathcal{M}$. By Theorem 4, $\mathcal{M}_3$ is compact. Moreover, Lemma 3 implies that $T(\mathcal{M}_3) \subset \mathcal{M}_3$.

Lemma 4. *The operator T is continuous on $\mathcal{M}_3$ (in the topology of uniformly convergence on compact subsets of R).*

Proof. Let $\{p^k\}_{k=1}^{\infty} \in \mathcal{M}_3$ such that $p^k \to \bar{p}$ as $k \to \infty$. Clearly, $\bar{p} \in \mathcal{M}_3$. We shall prove that $T(p^k) \to T(\bar{p})$ as $k \to \infty$.

Since $\left\{T(p^k)\right\}_{k=1}^{\infty}$ is precompact, it suffices to show that if a subsequence $\{T(p^{k_s})\}$ converges to $\tilde{p}$ then $\tilde{p} = T(\bar{p})$. To this end, let us consider the systems

$$(2.4_{k_s})\quad \dot{z}_i = z_i f_i(t, p_1^{k_s}(t), ..., p_{i-1}^{k_s}(t), z_i, p_{i+1}^{k_s}(t), ..., p_n^{k_s}(t)), \quad i = 1, ..., n$$

and

$$(2.5)\qquad \dot{z}_i = z_i f_i(t, \bar{p}_1(t), ..., \bar{p}_{i-1}(t), z_i, \bar{p}_{i+1}(t), ..., \bar{p}_n(t)), \quad i = 1, ..., n.$$

Clearly, the right hand side of (2.4_{k_s}) converges to the right hand side of (2.5) uniformly on any compact subset of $R \times R_+^n$. Therefore, from [5, Theorem 2.4, p. 4] it follows that $\tilde{p}(.)$ is a solution of (2.5). Since (2.5) has a unique solution in $\mathcal{C}^+$ (by Lemma 1), $T(\bar{p}) = \tilde{p}$. $\square$

Proof of Theorem 1.
(i) *The existence.* By Lemma 4 and Tychonov's fixed point theorem, there exists $x^* \in \mathcal{M}_3$ such that $T(x^*) = x^*$. Thus $x^*(t)$ is a solution of system (1.1) whose components are bounded above and below by positive constants.
(ii) *The permanence.* First of all we prove the following claim.

Claim. If $x(.)$ is a solution of (1.1) and $x_i(t_0) > 0$ for some $i \in \{1, ..., n\}$, then $x_i(t) > 0$ for all $t \geq t_0$ where the solution is defined.

Indeed, if $x_i(t_1) = 0$ for a minimal value $t_1 > t_0$, we have that $w^1(.) = x_i(.)$ and $w^2(.) = 0$ both are solutions of the scalar differential equation

$$\dot{w} = wf_i(t, x_1(t), ..., x_{i-1}(t), w, x_{i+1}(t), ..., x_n(t))$$

satisfying the same condition at t_1. Hence, for $t \in [t_0, t_1]$, we have

$$\begin{aligned}\Big|\frac{d}{dt}(w^1(t) - w^2(t))\Big| &= |w^1(t) f_i(t, x_1(t), ..., x_n(t))| \leq \gamma |w^1(t)| \\ &= \gamma |w^1(t) - w^2(t)|,\end{aligned}$$

for a suitable constant $\gamma > 0$. Since $w^1(t_1) - w^2(t_1) = 0$, the Gronwall lemma gives $w^1(t) - w^2(t) = 0$ for all $t \in [t_0, t_1]$, which contradicts the fact that $w^1(t) - w^2(t) > 0$ for $t \in [t_0, t_1)$. The claim is proved.

Let $x(.)$ be any noncontinuable solution of (1.1) with $x_i(t_0) > 0$, $(i = 1, ..., n)$. By the above claim and hypothesis (H_3), $x(.)$ is defined in $[t_0, +\infty)$ and $x_i(t) > 0$ for all $t \geq t_0$, $i = 1, ..., n$. For each $i = 1, ..., n$, let $u_i(.)$ be the (right) noncontinuable maximum solution of the scalar equation (2.2_i) with $u_i(t_0) = x_i(t_0)$. By (H_3), the relation

$$\dot{x}_i(t) = x_i(t) f_i(t, x_1(t), ..., x_n(t)) \leq x_i(t) f_i(t, 0, ..., 0, x_i(t), 0, ..., 0),$$

holds for each i and $t \geq t_0$. By [8, Lemma 2.6, p.318], we have

$$x_i(t) \leq u_i(t) \ (t \geq t_0, \ i = 1, ..., n). \tag{2.6}$$

Since $f_i(t, x)$ $(i = 1, ..., n)$ is uniformly continuous, the hypothesis (H_4) implies that there exists $\epsilon > 0$ such that, for $i = 1, ..., n$, it holds

$$\inf_{t \in R} f_i(t, X_1^0(t) + \varepsilon, ..., X_{i-1}^0(t) + \varepsilon, 0, X_{i+1}^0(t) + \varepsilon, ..., X_n^0(t) + \varepsilon) > 0. \tag{2.7}$$

By Lemma 1, $\lim\limits_{t \to +\infty} |u_i(t) - X_i^0(t)| = 0$, $(i = 1, ..., n)$. Hence (2.6) implies that there exists $t_1 \geq t_0$ such that

$$x_i(t) \leq X_i^0(t) + \varepsilon, \quad t \geq t_1, \ i = 1, ..., n. \tag{2.8}$$

By Lemma 1, for each $i = 1, ..., n$, the scalar differential equation

$$\dot{z}_i = z_i f_i(t, X_1^0(t) + \varepsilon, ..., X_{i-1}^0(t) + \varepsilon, z_i, X_i^0(t) + \varepsilon, ..., X_n^0(t) + \varepsilon) \tag{2.9_i}$$

has a unique solution $Z_i^0(t)$ which is bounded above and below by positive constants.

Let $z_i(.)$ $(i = 1, ..., n)$ be the (right) noncontinuable minimum solution of the scalar equation (2.9_i) with $z_i(t_1) = x_i(t_1)$. We have, for $t \geq t_1$,

$$\dot{x}_i(t) \geq x_i(t) f_i(t, X_i^0(t) + \varepsilon, ..., X_{i-1}^0(t) + \varepsilon, x_i(t), X_{i+1}^0(t) + \varepsilon, ..., X_n^0(t) + \varepsilon).$$

Consequently, by [8, Lemma 2.7, p.319] we have

$$z_i(t) \leq x_i(t),\ t \geq t_1,\ i = 1, .., n.$$

Once again, by Lemma 1, $\lim\limits_{t \to +\infty} |z_i(t) - Z_i^0(t)| = 0$, $(i = 1, ..., n)$.

We set

$$M = \varepsilon + \max_{1 \leq i \leq n} \Big\{ \sup_{t \in R} X_i^0(t) \Big\}, \quad m = \min_{1 \leq i \leq n} \Big\{ \inf_{t \in R} Z_i^0(t) \Big\}.$$

Then (1.3) holds, i.e., system (1.1) is permanent. The proof of Theorem 1 is now complete. □

Proof of Theorem 2. For $i = 1, ..., n$, let us set $\varepsilon_i = \gamma_i / 4$ and

$$\gamma_i = \lim_{\omega \to \infty} \frac{1}{\omega} \int_0^\omega f_i(t, X_1^0(t), ..., X_{i-1}^0(t), 0, X_{i+1}^0(t), ..., X_n^0(t)) dt > 0.$$

Since $f_i(t, X_1^0(t), ..., X_{i-1}^0(t), 0, X_{i+1}^0(t), ..., X_n^0(t))$ $(i = 1, ..., n)$ is almost periodic, the approximation theorem [7, p. 17] implies that there exists a trigonometric polynomial $\Delta_i(t)$ such that

$$\sup_{t \in R} |f_i(t, X_1^0(t), ..., X_{i-1}^0(t), 0, X_{i+1}^0(t), ..., X_n^0(t)) - \Delta_i(t)| < \varepsilon_i.$$

Clearly, we have that

$$\gamma_i - \varepsilon_i = \frac{3\gamma_i}{4} < \bar{\Delta}_i := \lim_{\omega \to \infty} \int_0^\omega \Delta_i(t) dt \leq \varepsilon_i - \gamma_i = \frac{5\gamma_i}{4}$$

and $\exp\{\bar{\Delta}_i t - \int_0^t \Delta_i(s) ds\}$ is almost periodic.

By a change of the variable

$$u_i = x_i \exp\Big\{\bar{\Delta}_i t - \int_0^t \Delta_i(s)ds\Big\} \quad (i = 1, ..., n)$$

the system (1.1) becomes

$$\dot{u}_i = u_i F_i(t, u), \quad i = 1, ..., n, \tag{2.10}$$

where

$$F_i(t,u) = f_i\Big(t, u_1 \exp\big\{-\bar{\Delta}_1 t + \int_0^t \Delta_1(s)ds\big\}, ..., \\ u_n \exp\big\{-\bar{\Delta}_n t + \int_0^t \Delta_n(s)ds\big\}\Big) + \bar{\Delta}_i - \Delta_i(t).$$

Hence, it suffices to show that system (2.10) is permanent and it has at least one solution defined on whole R, whose components are bounded above and below by positive constants.

It is clear that

$$\begin{aligned} F_i(t,\theta) &= f_i(t,\theta) + \bar{\Delta}_i - \Delta_i(t) \\ &\geq f_i(t, X_1^0(t), ..., X_{i-1}^0(t), 0, X_{i+1}^0(t), ..., X_n^0(t)) \\ &\quad + \bar{\Delta}_i - \Delta_i(t) \geq \frac{\gamma_i}{2} > 0. \end{aligned}$$

Therefore (by Lemma 1) $U_i^0(t) := X_i^0(t) \exp\big\{\bar{\Delta}_i - \int_0^t \Delta(s)ds\big\}$ is a unique solution in $\mathcal{C}^+$ of the following equation

$$\dot{u}_i = u_i F_i(t, 0, ..., 0, u_i, 0, ..., 0) \ \ (i = 1, ..., n).$$

Moreover,

$$\begin{aligned} &F_i(t, U_1^0(t), ..., U_{i-1}^0(t), 0, U_{i+1}^0(t), ..., U_n^0(t)) \\ &= f_i(t, X_1^0(t), ..., X_{i-1}^0(t), 0, X_{i+1}^0(t), ..., X_n^0(t)) + \bar{\Delta}_i - \Delta_i(t) \\ &\geq \frac{\gamma_i}{2} > 0. \end{aligned}$$

Thus system (2.10) satisfies all the conditions of Theorem 1. The proof of Theorem 2 is now complete. □

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